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ABSTRACT

The modal parameters, natural frequencies and modal damping ratios, of a 20 m high steel lattice mast subjected to natural excitation have been numerically and experimentally investigated. The undamaged as well as the damaged mast has been considered. For the damaged mast eight different damage scenarios were considered. In these damage scenarios a damage is assumed in one of the eight lower diagonals. These diagonals were cut and provided with a bolted joint. Each bolted joint consists of 4 slice plates giving the possibilities of simulating a 1/4, 1/2, 3/4 and full reduction of the sectional area of a diagonal. Thus, a damage was simulated by removing one or more splice plates in these bolted joints. Based on 20 periodical measurements during 6 months the aim of this report is to investigate the sensitivity of the modal parameters, identified by an ARMA-model, to environmental conditions such as wind-direction, wind-speed and air-temperature. These sensitivities are compared with the changes of modal parameters due to damage. It is found that the modal parameters are sensitive to the environmental conditions. During the 6 months, the measured natural frequencies vary only less than one per cent while the measured modal damping ratios vary more than ten per cent due to different environmental conditions. Further, it is found that the measured bending natural frequencies and the measured rotational frequency approximately decrease few per cent and more than ten per cent, respectively, due to a damage corresponding to a removal of one of the lower diagonals. This means that it is possible to detect such a damage in the mast using a system identification technique based on natural excitation. A damage corresponding to a fifty per cent reduction of the sectional area can also be detected.

PREFACE

The present report *An Experimental Study of the Modal Parameters of a Damaged Steel Mast* has been prepared as a part of the research project *In-Field Vibration Based Inspection of Civil Engineering Structures* which has been performed as a pilot project by the consulting firm Rambøll, Hannemann and Højlund (RH&H) in cooperation with the Department of Building Technology and Structural Engineering, University of Aalborg during the period from January 92 to July 93.

The figures have been carried out by the draughtsman Miss Helle Winkler from RH&H and the draughtsman Mrs. Norma Hornung from University of Aalborg. The proof-reading has been performed by secretary Mrs. Solveig Hesselvang from University of Aalborg. Their carefully performed work is greatly appreciated.

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Aalborg, Denmark
July, 1993

Anders Rytter
Poul Henning Kirkegaard

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1. INTRODUCTION

Structural diagnosis (health monitoring) by measuring vibrational signals of civil engineering structures is a subject of research which has received increasing interest during the last decades. The main impetus for doing vibrational based inspection (VBI) is caused by a wish to establish an alternative damage assessment method to the more traditionally methods. The most common of the traditional methods is visual inspection. However, damage assessment by visual inspection can be costly, risky and difficult when civil engineering structures such as offshore structures, bridges etc. are considered. Besides, a reduction of inspection cost a capable VBI technique can lead to less risky and a quick means of assessing structural damage. Many research projects have concluded that it is possible to detect damages in civil engineering structures by VBI and some techniques to locate damages in civil engineering structures have also been proposed. However, much of the performed research has been based on using numerical simulations and/or laboratory models. A throughout review of VBI techniques can be found in Rytter [1].

In order to use VBI techniques it is necessary to be able to obtain reliable estimates of the dynamic characteristics, e.g. natural frequencies. Such quantities can be estimated from the resulting output caused by a known well-defined input. However, the estimates can also be estimated by using the so-called ambient testing, i.e. the only excitation on the structure is the natural excitation. Normally, only the output and not the input is measured during ambient testing.

The aim of the research presented in this report was to investigate the possibility of detecting and locating damages in civil engineering structures, by using full-scale measurements based on natural excitation, i.e. ambient testing. The capacity of a time domain identification method (ARMA) to detect small but structurally significant changes was investigated. An important step of the investigations was to compare the precision of the modal parameters obtained by the time domain technique and the deviation in the modal parameters due to damages. Since VBI is often periodic at intervals in the order of several months non-structural changes in the physical conditions such as mass redistribution etc. could influence the dynamic characteristics. Further, one could also expect that the results would be influenced by changes in the ambient environmental conditions. I.e. the aim of the research was to answer the following questions:

1) Is it possible to distinguish between effects produced by damages and those brought about as a result of changes in the ambient environmental conditions?

2) Assuming 1) above can be satisfactorily solved, how sensitive are vibration responses then to a damage?

3) Is it possible to locate damages from modal parameters estimated by using natural excitation?

In order to answer these questions a 20 m high steel lattice mast subjected to wind excitation was numerically and experimentally investigated. The experimental arrangement is described in section 2. In section 3 an numerical analysis (FEM) of the steel mast is performed. The time domain system identification method (ARMA) is described in section 4 and the experimental results are presented and discussed in section 5.

2. EXPERIMENTAL ARRANGEMENT

The structure considered is a 20 m high steel lattice mast. Originally, the mast was used as one of the legs in a portal transmission mast. The four chords K-frame test mast with a 0.9x0.9 m cross-section was bolted with twelve bolts, three for each chord, to a concrete foundation block founded on chalk and covered by sand near the Structural Research Laboratory of Department of Building Technology and Structural Engineering, University of Aalborg. This implied that the data acquisition equipment could be placed inside the laboratory. A site plan is shown in enclosure A.

2.1 Mast-Undamaged

An elevation of the test mast is shown in enclosure A. The chords as well as the diagonals of the steel lattice mast are L-profiles with the dimensions given in enclosure A. The mast was constructed with welded connections. At the top of the mast two plywood plates were placed, as shown in enclosure A, in order to increase the wind-area. The total weight of the mast is approximately 2000 kg.

2.2 Mast-Damaged

Eight different damage scenarios were considered. In these damage scenarios damage was assumed in one of the eight lower diagonals denoted AB101, AB102, BC101, BC102, CD101, CD102, DA101, DA102, respectively, see figure 2.1.

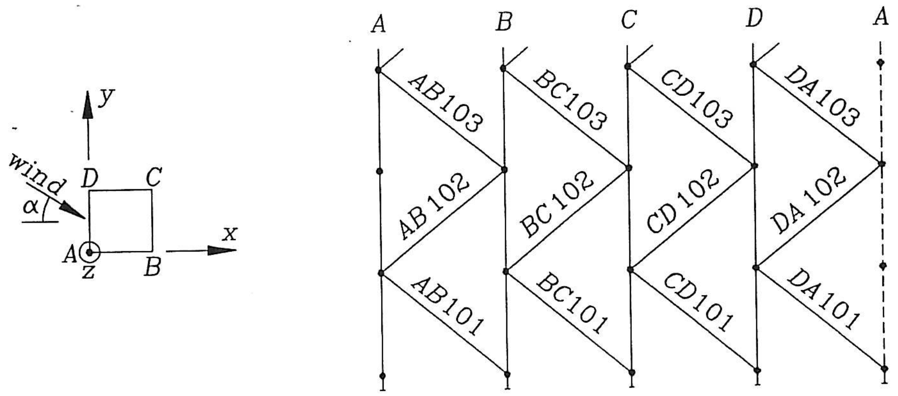

Figure 2.1: Diagonals of the lower two sections of the mast.

The eight lower diagonals were cut and provided with a bolted joint. Each bolted joint consists of 4 slice plates giving the possibilities of simulating a 1/4, 1/2, 3/4 and full reduction of the area of a diagonal, see enclosure A. A damage was simulated by removing one or more splice plates in these bolted joints.

2.3 Instrumentation

Enclosure B shows the diagram of the instrumentation of the mast used to measure the accelerations, wind-direction (wind-vane) and wind-speed (cup-anemometer). The readings of these instruments were recorded on magnetic tape using a KYOWA RPT-600B 16 channels tape-recorder, see KYOWA [2]. Further, the ambient air temperature was measured. The cables connecting accelerometers, cup-anemometer and wind-vane with the data acquisition equipment were 7-pin cables. All other connections were made with BNC-cables.

2.3.1 Accelerometers

The horizontal acceleration response of the mast was measured with 3 mechanical Schaevitz accelerometers, type LSBC-1 (1.0 g acc.) placed at the top of the mast and 3 mechanical Schaevitz accelerometers, type LSBC-0.5 (0.5 g acc.) placed at the middle of the mast, see Schaevitz[3]. The accelerometers had a built in amplifier.

Enclosure A shows how the accelerometers were mounted at the mast. The accelerometers were horizontally located. Further, the accelerometers were placed in accelerometer houses in order to protect against the environment. It is seen from enclosure A that the accelerometers 1.1, 1.3, 2.1 and 2.3 were arranged in parallel to each other and in 90 degrees to the accelerometer 1.2 and 2.2.

2.3.2 Cup-Anemometer and Wind-Vane

A cup-anemometer and a wind-vane to measure the speed and direction of the wind, respectively, were placed at the top of the mast. The wind-speed was measured with a one propeller Carl Th. Malling cup-anemometer, type 882.312, see Malling [4]. By calibration of the cup-anemometer it was found that the output exit potential range corresponds to 8 m/s per voltage. The wind direction was measured with a Carl Th. Malling vane, type 884.312, see Malling [4] with a recording range 0-360 degrees corresponding to an output potential range 0-4.95 voltage.

2.3.3 Data Acquisition

The data acquisition system used in the project was based on a personal computer 386-40 MHz. with an add-on 16 A/D data acquisition board, type DT2829, see Data Translation [5]. Before sampling the measured acceleration signals were analog low-pass filtered with a 8-pole Butterwoth filter, type Rockland serie 2000, model 2382/2582, see Rockland[6]. The signals were amplified by the tape-recorder in order to reduced the quantization error. The data acquisition and the analyse of the sampled data were performed with a program to Structural Time Domain Identification, STDI, see Kirkegaard et al. [7].

3. FINITE ELEMENT ANALYSES

A finite element analysis of the undamaged as well as the damaged mast was performed. The aim of the analyses were

- to obtain estimates for the mode shapes and the corresponding natural frequencies
- to evaluate the safety of the mast when loaded with wind load

The model consisted of 249 linear elastic beam elements. The modullus of elasticity and the Poisson ratio were taken as $2.1 \cdot 10^5 N/mm^2$ and 0.3, respectively. The density of the steel was 7850 kg/m^3. The mass of the single element was generated by the programme. All the beams were fixed against translation and rotation in the joints. The three bolts at each of the four fixtures of the mast were equalized by a beam element. The elements were denoted in accordance with the coding diagram shown in enclosure C.

Eight different damage scenarios were considered. In these damage scenarios a damage was assumed in one of the eight lower diagonals, which were denoted AB101, AB102, BC101, BC102, CD101, CD102, DA101 and DA102, respectively, see figure 2.1. The damage state was characterized by a number between 0 and 1 corresponding to the actual cross sectional area of the damaged area, i.e. 0 and 1 correspond to a total collapsed element and an intact element, respectively. The analyses have been performed by means of the computer package ROSAP, see ROSAP [8].

3.1 Modal Analysis

The six lower natural frequencies of the mast were chosen as potential damage indicators. These natural frequencies have been estimated by means of the FE-programme ROSA from the computer package ROSAP, see ROSAP [8], for the intact mast and total damage in the 8 elements inherent in the damage scenarios, see table 2.1. Frequencies no. 1 and 4 correspond to bending parallel to the x-axis, no. 2 and 5 correspond to bending parallel to the y-axis and frequency no. 3 and 6 correspond to rotation around the vertical centre line of the mast. Plots of the mode shapes for mode 1-3 are shown in enclosure D.

The results in table 3.1 show that significant changes in the measured natural frequencies should be expected as one of the eight lower diagonals is removed. The rotational frequencies (frequency no. 3 and 6) are the most sensible. Further, it can be seen that the natural frequencies of the bending modes parallel to the x-axis and y-axis, respectively, do not change when the damage is introduced in a diagonal perpendicular to the actual axis. Thus, the natural frequencies no. 1 and 4 which correspond to deflections parallel to the x-axis are affected in damage states no. 1, 3, 5 and 7 only, whereas the natural frequencies no. 2 and 5, which correspond to deflections parallel to the y-axis, are affected in the damage states no. 2, 4, 6 and 8.

Nat. Freq. No.	Damage State								
	0	1	2	3	4	5	6	7	8
1	2.012	1.043	2.012	1.937	2.012	1.921	2.012	1.913	2.012
2	2.018	2.018	1.944	2.018	1.949	2.018	1.920	2.018	1.926
3	8.184	6.011	6.163	6.123	6.081	5.559	5.174	5.674	5.613
4	11.528	10.389	11.528	10.165	11.530	10.345	11.530	10.111	11.531
5	11.622	11.622	10.179	11.621	10.422	11.623	10.121	11.622	10.372
6	26.982	19.851	20.076	20.095	19.647	20.153	20.392	20.415	19.925

Table 3.1: The six lower natural frequencies (Hz.) of the mast for the undamaged and damaged mast.

Damage State no. 0: Undamaged mast
Damage State no. 1: Elem. AB101 removed
Damage State no. 2: Elem. BC101 removed
Damage State no. 3: Elem. CD101 removed
Damage State no. 4: Elem. DA101 removed
Damage State no. 5: Elem. AB102 removed
Damage State no. 6: Elem. BC102 removed
Damage State no. 7: Elem. CD102 removed
Damage State no. 8: Elem. DA102 removed

3.2 Safety Check

The utility ratios (abbreviated URs) of the different elements of the mast have been calculated by means of the post-processor programme STRECH, see ROSAP [8], in accordance with the Danish codes for safety and loads, DS409 and DS410 [9] and steel structures DS412 [10]. The URs are defined as the ratio between the actual stress and the allowable stress in a member. Thus, the safety is in order when the URs are less than or equal to 1.

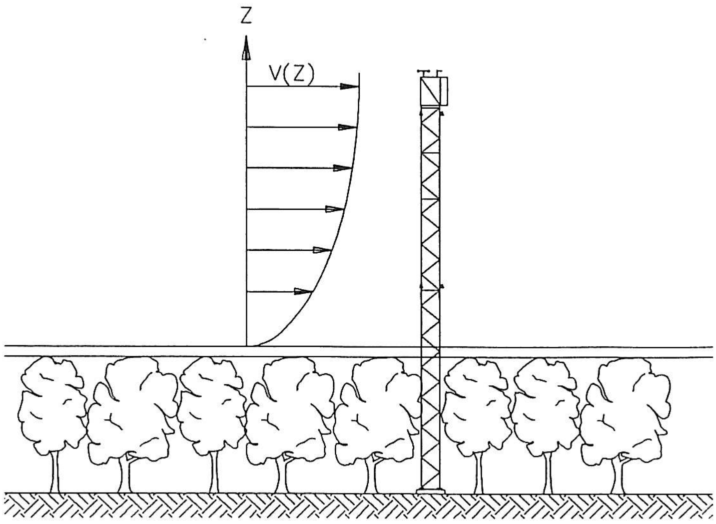

Figure 3.1: Wind Load on Mast.

Dead loads and wind loads are the sole basic loads, which have been included in the performed analysis. The mast is placed between trees and near the buildings of the University of Aalborg, see enclosure A. Therefore, the terrain class 0.3, see DS410 [9] has been used. Further, in accordance with Dyrbye et al. [11], the trees have been included in the boundary layer for the wind, see figure 3.1.

The dynamic effects of the wind is in accordance with DS410 [9] taken into account by multiplying the static wind load by a gust factor of 2.85. Eight different combined load cases corresponding to α = i*45, i=0,1,2,..,7 , see figure 2.1 have been included.

The maximum URs for each of the 8 damage states defined in table 3.1 are shown in table 3.2.

Damage State	Maximum UR	Comments
0	0.52	
1	0.89	
2	0.89	
3	0.89	
4	0.89	
5	1.10	The high URs occur due to global buckling of the
6	1.12	element AB101, BC101, CD101 and DA101, respectively.
7	1.10	The maximum URs decrease to about 0.92, if these
8	1.12	elements are removed from the FEM

Table 3.2: Maximum utility ratios.

It can be seen from table 3.2 that a total collapse of one of the 8 lower diagonals is an acceptable level of damage in the mast.

3.3 Conclusion

The performed analysis has shown that a total collapse of one of the 8 lower diagonals is an acceptable level of damage in the mast. Further, the analysis has shown that these damage scenarios cause a decrease of about 0.06 Hz in one of the two fundamental bending frequencies depending on the location of the damage. A decrease of 2.0-2.6 Hz and 6.6-7.3 Hz has been estimated for the first and second rotational natural frequencies. The natural frequencies of the second bending mode parallel to the x-axis and y-axis, respectively, either decrease about 1.5 Hz or remain unchanged depending on the location of the damage.

Thus, from the results of the performed analysis it may be reasonable to demand that the standard deviation of the estimates for the six lower natural frequencies obtained by means of the SI-method should be of the order 0.025 Hz. If this demand is not fulfilled, then the two fundamental natural frequencies should either be included with a lower weight or excluded in the diagnostic session. Further, it should be requested that the level two or three method(-s) to be used in the diagnostic session should manage to reveal a total collapse in one of the eight lower diagonals.

4. SYSTEM IDENTIFICATION

In this chapter it is described how the modal parameters can be estimated by an ARMA-model based on ambient vibration testing. In section 4.1 some general remarks concerning system identification are given and in section 4.2 time domain versus frequency domain system identification is discussed, and the principle of system identification by using an ARMA-model is described.

4.1 General Remarks

System identification is a process for determining the relationship between cause and effect in a physical system. More specifically, if one is given the input and the corresponding output for a system, whose description is unknown, the goal of system identification is to find a mathematical model that describes the system. In general, it is found that system identification is limited to parameter estimation in a model chosen a priori. System identification techniques have been widely used in many branches of science and engineering for the estimation of various characteristics of a physical system, see e.g. Eykoff [12]. Applications of system identification in civil engineering have been studied with increasing interest during the last two decades. Primarily motivated by the desire to have a more accurate description of the structure and its dynamic characteristics for the purposes of predicting its response to environmental excitations such as earthquake, wave and wind generated pressure loads. Further, to assess safety or damage through changes in structural parameters and finally for the purposes of applying controllers to structures that can reduce environmental excitations. Certainly, for these reasons and, perhaps, others that one may think of, an accurate model of the real structure must be available to the engineer. In general, it can be stated that the aims of structural identification are several where models and techniques differ depending on the aims. Different aims of structural identification are discussed in e.g. Natke et al. [13], Natke [14], Hart et al. [15], Ibanez [16], Aktan et al. [17] and Vestroni et al. [18].

Historically, system identification based on frequency domain models seemed to dominate the theory and practice of the system identification up to the sixties. From the end of the sixties and onwards the interest in the system identification based on time domain models has increased and now literature on system identification is very much dominated by time domain methods. System identification based on time domain models and frequency models, respectively, can be considered as two complementary approaches. System identification based on frequency models often gives good insight into the properties of the data and the system. Such information combined with engineering intuition is valuable for decisions of type and complexity of models to be used for further analysis of the data. Therefore, most of the frequency domain approaches can be used as a good first step in the data analysis procedure. Often the intended use of the model as well as accuracy requirements on parameter estimates motivate the use of a time domain model and corresponding system identification procedure. In Ljung et al. [19] the basic features of system identification based on time and frequency domain approaches are highlighted. Further, relationships between the two approaches are explained. Time domain approaches versus frequency domain approaches are also discussed in e.g. Prevosto et al. [20] and Davies et al. [21]. A more thorough outline can e.g. be found in recent survey papers such as Kozin et al. [22]. Comprehensive surveys showing the trend in structural identification are also given in e.g. the following Ph.D. theses: Ahmadi [23], Jayakumar [24], Sprandel [25] and Jensen [26]. Further, it is possible to find the latest developments in system identification techniques, used in civil engineering problems, in the proceedings of national and international conferences such as the IMAC (International Modal Analysis Conference), and the annual seminars in Hannover and Leuven.

4.1.1 Structural Modelling

Civil engineering structures can be regarded as distributed parameter systems characterized by distribution of the mass, damping and stiffness properties. However, parameter identification in distributed parameter systems is generally not easy. Thus, with a few exceptions, in most of the literature on testing of structures, the data are analysed based on the assumption that the system is described by one or a set of linear ordinary second-order differential equations. Because of their simplicity the linear time invariant lumped parameter models are the most widely used models in structural identification. More complex models such as the linear continuous parameter models and non-linear models are used only when the lumped-parameter model cannot be used to provide an adequate representation of the structural behaviour. Since a model is chosen a priori in the structural system identification the system identification problem in civil engineering is then reduced to the estimation of the parameter measurements.

Parameter estimation of structures is usually greatly simplified, as mentioned above, by certain assumptions of the structures, e.g. that they are linear in the dynamics, time invariant and that they can be adequately described by a discrete mass-spring-damper model

$$\overline{\overline{M}}\ddot{\overline{y}}(t) + \overline{\overline{C}}\dot{\overline{y}}(t) + \overline{\overline{K}}\overline{y}(t) = \overline{u}(t) \tag{4.1}$$

where $\overline{\overline{M}}$, $\overline{\overline{C}}$ and $\overline{\overline{K}}$ are the mass, damping and stiffness matrices. $\overline{y}(t)$ and $\overline{u}(t)$ are the displacement and force vectors at the nodes, respectively. Here, it is assumed that the distributed forces of inertia of the structure can be discretized into n degrees-of-freedom and be given by $\overline{\overline{M}}\ddot{\overline{y}}(t)$. This set of forces of inertia is balanced by a set of linear-elastic restoring forces $\overline{\overline{K}}\overline{y}(t)$, viscous damping $\overline{\overline{C}}\dot{\overline{y}}(t)$ and the external loads $\overline{u}(t)$. Assuming a linear structure means that the response of the structure to any combination of forces, simultaneously applied, is the sum of the individual responses to each of the forces acting alone. This is a good assumption for a variety of structures. The time invariant assumption implies that the parameters are to be determined as constants.

The basic goal of parameter estimation is to find the $\overline{\overline{M}}$, the $\overline{\overline{C}}$ and the $\overline{\overline{K}}$ matrices from measured values of $\overline{y}(t)$ and $\overline{u}(t)$ not necessarily being at the same locations. The external forces may be zero if the structure vibrates freely due to an applied velocity and/or initial displacement. Thus, the parameters to be identified consist of physical/geometrical parameters. Identification of physical/geometrical parameters is the most straightforward approach for identification of structural systems. The advantage of identification of those parameters rather than the modal quantities is that the engineer may have some a priori knowledge about the physical/geometrical parameters. However, in general, the identification techniques determine the so-called modal parameters (modal damping, natural frequencies, mode shapes). Therefore, the term "experimental modal analysis", is often used instead of "system identification". However, it may be noticed that the term "experimental modal analysis" has earlier been used for one specific parameter estimation method based on so-called frequency-response functions, where the modal parameters are obtained by a curvefit of the frequency-response function of the model to that of the test data, see e.g. Ewins [27] for details. Nowadays, "experimental modal analysis" is synonymous with "system identification" when modal parameter identification problems are considered.

4.1.2 Ambient Excitation

From an experimental point of view the simplest approach to measure the dynamic parameters of a civil engineering structure is to detect the response to natural forces such as those caused by wind or waves. Such natural loads are called ambient excitation and the vibrations of the structure caused by them are called ambient vibrations. Another source of dynamic excitation for system identification of civil engineering structures can be artificial excitation.

The ambient excitation is random in nature. Therefore, it cannot be described by an explicit function with time and its characteristics are described by certain statistical parameters, such as mean and standard deviation. This means that the response of the structure is also random and may also be represented by its statistical characteristics. However, the statistical parameters of the response are different from those of the loading. These differences represent the effect of the structure.

Ambient excitation has been shown to be inexpensive, quick and reliable for testing of civil engineering structures such as buildings and offshore structures, see e.g. Ibáñez [28], Srinavasan et al. [29], Rubin et al. [30] and Jensen [31]. In Jensen [31] an extensive survey of the available literature concerning full-scale measurements on offshore platforms has been performed. It is found that the typical excitation of offshore platforms for system identification is ambient excitation. Further, Morgan et al. [32] concludes, based on a study of several published results of ambient versus forced vibration tests of high-rise structures in USA, that parameter estimates obtained by ambient excitation are as good as parameter estimates obtained by artificial excitation.

It is a disadvantage of ambient excitation tests that the characteristics of the input dynamic forces on the structure cannot be controlled and measured directly, i.e. the input cannot be quantified as to amplitude, spectral content or points of application to the structure. When ambient excitation is measured the observations of the excitation are often given as e.g. time series of the sea surface elevation if the system identification of an offshore structure is considered. From the time series characteristics of the sea states, such as significant wave height and average zero upcrossing period, can be estimated. These parameters can then be used as input to models, wave theories, which have been developed to describe waves either as time series or as spectra. The connection between the theoretical description of the waves and the forces on the structure is established using a load model. Such a model is e.g. the well-known Morison equation. A more thorough discussion of the theory used to establish the connection between observations of the waves and the forces on the structure can be found in e.g. Sarpkaya et al. [33].

Instead of measuring the ambient excitation it may often be assumed that these ambient excitations are white in spectrum, at least on a limited frequency band, i.e. a white noise approximation of the input is used for the identification of the structure under consideration. This assumption is used in ARMA-model based system identification.

If natural loads are not available or insufficient for dynamic tests of a civil engineering structure under investigation, the test structure has to be excited by a suitable chosen test device. Which kind of excitation (exciters) is optimum depends on the test object as well as the time available for carrying out the test. For dynamic tests of civil engineering structures where transient response is desired it is often useful to apply step relaxation (snapback testing) or impulse load. In snapback testing the structure is preloaded with a measured static force through a cable that is suddenly released causing the structure to undergo free vibrations. Snapback testing is convenient if e.g. high amplitude excitation or low frequency excitation is required. It is also useful when an access for mounting a vibration shaker is limited. Snapback testing is subject to certain limitations in practice. With this excitation technique in general only the lowest natural modes of the structure are sufficiently excited. This disadvantage can be overcome by repeated excitation applied at various locations. Besides, snapback testing impulsive loads can also be used to provide a transient response.

A survey of artificial excitation signals for dynamic system identification of civil engineering structures is given in e.g. Ewins [27], Schoukens et al. [34] or Natke et al. [35].

4.2 System Identification by an ARMA-Model

System identification based on frequency models often gives good insight into the properties of the data and the system. Therefore, a frequency domain approach can be a good first step in the data analysis procedure. Often the intended use of the model as well as the accuracy requirements for parameter estimates motivates the use of a time domain model.

The most frequent way of estimating modal parameters is in the time domain, where the free vibration response can be analysed by the so-called logarithmic decrement method to obtain a damping estimate and the zero-crossing period can be measured to estimate the natural frequency.

In the frequency domain the natural frequency can be estimated by direct examination of e.g. the peaks of the response spectra or transfer functions if the Fourier transform is used. If it is assumed that the damping ratio is small, i.e. less than 0.05, it is easy to show that the damping ratio is related to the half-power bandwidth.

Such direct techniques, as mentioned above, are only one-parameter-estimation techniques and may fail to work in many situations; other, more systematic ways of fitting the data, may be needed to estimate the parameters. This is particularly true when the data measurements are encumbered with noise.

A simple curvefit method available for estimating the modal parameters of a single-degree-of-freedom system is the single-degree-of-freedom curve-fit or often the circle-fitting method, see Ewins [27], which is a frequency domain method. The method utilises the fact that when the damping is small the Nyquist plot approximately traces a circular arc around the natural frequency. A Nyquist plot is obtained by plotting the real and imaginary part of a transfer function against each other for the given frequency range. For a multi-degree-of freedom system a Nyquist plot can be plotted for each resonance frequency. However, if the modes are clustered the method becomes inadequate since the circle-fit method is based on a single-degree-of-freedom assumption and therefore assumes that near a resonance, the behaviour of most systems is dominated by a single mode. When the circle of each Nyquist plot is determined it is easy to estimate the damping from the plots by simple expressions, see Ewins [27].

The most commonly used curve-fitting method in the frequency domain is a parameter estimation method based on frequency-response functions, where the modal parameters are obtained by a curve-fit of the frequency-response function of the model to that of the test data, see e.g. Ewins [27] for details. In this method the frequency response functions are measured using excitation at single or multiple points.

The methods shortly mentioned above assume that the excitation is measured. In Rytter et al. [36] it is shown that it is possible to estimate the modal parameters by using a global curve-fit of the response spectrum of the model to that of the measured data. The idea in the approach is to make a system identification based only on measurements of the response and not of the excitation. In the approach the shape of the force spectrum can be parameterized and included as unknown in the estimation procedure. However, it is a very time-consuming approach.

The parameter estimation techniques mentioned until now have mainly been frequency domain techniques. However, the frequency domain techniques have several drawbacks. Firstly, rather long term records (data) are required to ensure a reliable frequency solution. Secondly, after producing e.g. structural frequency response functions the parameters are not readily available. In fact, curve fitting algorithms must be subsequently used for this purpose. Thirdly, when structures not lightly damped and with clustered modes are considered poor parameter estimates are usually obtained. Finally, it may be mentioned that spectral density functions estimated by Fourier transform techniques will always be biased. The bias in the frequency domain can be reduced but not removed, see e.g. Bendat et al. [37].

4.2.1 ARMA-Model

Through the quest for new identification techniques that could overcome the weakness of the Fourier techniques and become more suitable for digital processing, several time domain algorithms have been developed. Many of these algorithms seek to describe a dynamic system by means of an autoregressive-moving-average (ARMA) model. An ARMA(n, m) model of order n, m describing the response at the discrete time points y_t is given by

$$y_t = \sum_{i=1}^{n} \Phi_i y_{t-i} - \sum_{i=1}^{m} \mathcal{O}_i e_{t-i} + e_t \tag{4.2}$$

Φ_i is an Auto Regressive (AR) parameter, $\mathcal{O}_i$ is the Moving Average (MA) parameter and e_t is a time series of a white noise process. This model involves a difference equation in which the output of the system is expressed as linear combination of past output, as well as present and past input. This kind of model is particular well suited for identification and response calculation purposes since they provide efficient system representations. For many years the identification techniques based on ARMA models have attracted limited interest concerning structural engineering applications. A factor contributing to this situation is that ARMA models have been developed primarily by control engineers and applied mathematicians. Further, ARMA models have been primarily developed concerning systems for which limited a priori knowledge is available, whereas the identification of structural systems relies heavily on understanding of physical concepts.

In recent years the application of ARMA models to the description of structural systems has become more common, see e.g. Gersch et al. [38] Pandit et al. [39], Hac et al. [40], Natke [14] and Jensen [26]. The time domain identification techniques using ARMA representation have been compared with frequency domain techniques in e.g. Davies et al. [21]. In this and other papers it has been documented that these ARMA time domain modelling approaches are superior to Fourier approaches for the identification of structural systems. These foundings make identification techniques utilising ARMA algorithms interesting for modal parameter estimation.

It may be noticed that the ARMA models give a direct relation to the modal parameters while the Fourier methods give a non-parametric model which, followed by a curvefitting algorithm, give the estimates of the modal parameters. If an ARMA$(2n, 2n-1)$ model is used for a stationary Gaussian white noise excited linear n-degrees-of-freedom system it can be shown that the covariance of the response due to the ARMA-model and that of the white noise excited structure will be identical, see e.g. Kozin et al. [22]. In other words, an ARMA model will provide an unbiased estimate of the autospectrum provided the assumptions hold. It is seen that the parameter identification of civil engineering structures by using an ARMA model assumes that the response data are caused by a white noise input to the structure. However, for wave or wind excited ligthly damped civil engineering structures, this assumption will normally hold, see section 4.1.2.

The AR-parameters are obtained by minimizing an error function V_N expressing the variance of e_t

$$V_N = \frac{1}{N}\sum_{t=1}^{N}\epsilon_t^2 = \frac{1}{N}\sum_{t=1}^{N}\frac{1}{2}(y_t^M - \hat{y}_t)^2 \tag{4.3}$$

where N is the number of data and ϵ_t is the prediction error. y_t^M and $\hat{y}_t$ are the measured response and the predicted response by (4.2), respectively. It may be noticed that the white noise assumption must be checked when the AR and MA parameters and the residuals have been estimated. If the assumption does not hold it may indicate that the order of magnitude of the model is too low and therefore should be increased. When the AR parameters are estimated the dynamic parameters are found from the $2n$ roots, λ_i of the characteristic polynomial of the AR-parameters:

$$\lambda^{2n} - \Phi_1\lambda^{2n-1} - \cdots - \Phi_{2n-1}\lambda - \Phi_{2n} = 0 \tag{4.4}$$

In e.g. Pandit et al. [41] it is shown that the roots are related to the modal parameters through the $2n$ relations

$$\lambda_i = \exp(\mu_i\Delta t) \qquad i = 1, 2, .., 2n \tag{4.5}$$

where Δt is the sampling interval. μ_i has the following relation to the modal parameters for an undamped system

$$\mu_i = -\omega_i\zeta_i \pm i\omega_i\sqrt{1-\zeta_i^2} \qquad \zeta_i < 1.0 \tag{4.6}$$

By using the ARMA model all the information in the measured time series is used to estimate the AR-parameters. This implies that a large amount of data has to be handled in the system identification process implying that it can be time consuming to estimate the parameters. Especially, when the model order increases, caused of the non-linear optimization which has to be used to get the AR-parameters and the MA-parameters. However, Wolds [42] has shown that any ARMA model can be represented by an AR model if the model order is chosen sufficiently high. This implies that the AR-parameters can be estimated directly by linear regression obtaining a least squares fit between the measured time series and the AR-model. The AR-parameters can also be obtained from estimates of the auto-correlation function. If (4.2) is multiplied at both sides by y_{t-k} and then take the expectation a difference equation for the auto-correlation function is obtained. This gives a set of linear equations in the AR-parameters often referred to as Yule-Walker equations. When the number of equations exceeds the number of parameters to be estimated, the system becomes overdetermined, and the equations have no solution. For such situations, however, standard methods for determination of approximate solutions exist. One possibility is to solve the system of equations by least squares linear regression. In that case estimates of the AR-parameters are called overdetermined Yule-Walker estimates, see e.g. Söderström [43]. Using estimates of the auto-correlation function implies that the system identification process becomes less time-consuming since the auto-correlation function is estimated from a small amount of data compared to the original time series and because the estimates of the AR-parameters can be obtained by simple linear regression instead of optimization. In Brincker et al. [44] it is found, by investigation of a SDOF based on simulated data that this technique of modal parameter estimation is almost as accurate as calibration of an ARMA model directly on the original time series, but faster, especially if the estimates of the auto-correlation function are obtained by the Random Decrement Technique, see Brincker et al. [45].

4.2.2 Model Selection and Model Validation

Model selection involves the selection of the form and the order of the model, and constitutes the most important part of the system identification. Model validation is to confirm that the model estimated is a realistic approximation of the actual system. A throughout description of the problem of model selection and validation is given in e.g. Ljung [46] and Söderström [43]. In the following it will be shortly explained how one can deal with this problem. In general, the choice of the model structure involves:

- *Model type.* This involves the selection between non-linear and linear models, between black-box and physical models etc.
- *Model size.* I.e. choice of model order and number of adjustable parameters.
- *Model parameterization.* I.e. the way in which the number of parameters enter into the model.

The choice of the model to a large extent should be made according to the aim of the final purpose. There is no general solution of this problem but a large number of methods to assist in the choice of an appropriate model structure exist. These methods can be divided into several categories. They are based on

- *A priori knowledge.* Information about the system obtained from e.g. understanding of the physics of the system, design calculations, etc.
- *Preliminary data analysis.* Extracting information from the data that involve determination of a complete model of the system. E.g. spectral analysis estimates will give valuable information about resonance peaks. Further, a preliminary data analysis test for non-linear effects can be performed.
- *Comparison of model structures.* A most natural approach to search for a suitable model structure is simply to test a number of different ones and then to compare the resulting models. However, it is usually only feasible to do this with simple models because of the amount of calculation involved in more complicated models.

For such comparisons, as mentioned above a discriminating criterion is needed. The comparison of the model structures can be interpreted as a test for a significant decrease in the minimal values of the loss function V_N associated with the model structures in question. As a model structure is expanded, e.g. increasing the number of adjustable parameters, the minimal value of V_N decreases since new degrees of freedom have been added to the optimization problem. The decrease of V_N is a consequence that more flexible model structures give a possibility for better fit to the data. On the other hand when a good fit can be obtained there is no reason to increase e.g. the number of adjustable parameters. An overparameterized model structure, i.e. containing several models giving a perfect description of the actual system, can lead to unnecessarily complicated computations for finding the parameter estimates. An underparameterized model, i.e. a model having too few parameters to describe the system adequately, may be inaccurate. In order to deal with this problem Akaike, see Akaike [47], suggested a Final Prediction Error (FPE) criterion and a closely related Information Theoretic Criterion (AIC) of the type

$$FPE = \frac{1+\frac{n}{N}}{1-\frac{n}{N}} V_N \tag{4.7}$$

$$AIC = \log[(1+\frac{2n}{N})V_N] \tag{4.8}$$

where N is the length of the data record and n is the total number of estimated parameters. The model structure giving the smallest value of these criteria is selected. The AIC and FPE criteria penalize using too high model orders, i.e. their value may increase with increasing model order.

In e.g. Ljung [46] and Söderström [43] other approaches to model structure comparisons are given.

Model validation is the final stage of the system identification procedure. In fact model validation overlaps with model structure selection. Since the system identification is an iterative process various stages will not be separated: models are estimated and the validation results will lead to new models etc.

Model validation involves two basic questions:

- What is the best model within the chosen model structure ?
- Is the model fit for its purpose ?

One of the dilemmas in model validation is that there are many different ways to determine and compare the quality of the estimated models. First of all, the subjective judgement in model validation should be stressed. It is the user that makes the decision based on numerical indicators. The variance of the parameter estimates can be such an indicator. High values indicate a model with a bad fit or overparameterization. It is also important to check whether the model is a good fit for the data recording to which it was estimated. If it is a bad fit it may e.g. indicate that the model represents a local minimum. Simulation of the system with the actual input and comparing the measured output with the simulated model output can also be used for model validation. Statistical tests of the prediction errors ϵ_t are also typically used numerical indicators for model validation. If the statistical distribution of ϵ_t matches the assumed distribution then it can be concluded that the system dynamics is indeed well represented by the model. Any different trend in the statistical characteristics originally assumed is an indication that either the model or the noise is incorrectly assumed or that the parameters are incorrectly estimated.

The above-mentioned tools for model validation lead to a conclusion as to whether the model is fit for its purpose.

4.2.3 Estimation of Parameter Uncertainty

From measurements of the response process it is possible to get unbiased estimates of the AR-parameters Φ_1 and Φ_2, see e.g. Pandit et al. [41], where estimates of the variances of the estimated parameters can be estimated by the Cramer-Rao lower bound. This implies that the covariance matrix of parameter estimates can be obtained by the inverse of the Fisher information matrix $\overline{\overline{J}}$ which can be written

$$\overline{\overline{J}} = \frac{N}{\lambda_{\mathcal{E}}} E[\overline{\Psi}_t(\overline{\Phi})^T \overline{\Psi}_t(\overline{\Phi})] \tag{4.9}$$

A realization of the stochastic process $\{\overline{\Psi}_t(\overline{\Phi})\}$is given by

$$\overline{\psi}_t(\overline{\Phi}) = \frac{\partial \epsilon_t(\overline{\Phi})}{\partial \overline{\Phi}} \tag{4.10}$$

It is assumed that the variance of the noise process $\{\mathcal{E}_t\}$ is $\lambda_{\mathcal{E}}$. N is the number of samples. $\overline{\Phi}$ is a vector including the AR-parameters.

When the elements of the information matrix are calculated the parameter covariance matrix $\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$ of estimates of the parameter vector $\hat{\overline{\theta}}_N$ can be expressed in the following way

$$\overline{\overline{C}}_{\hat{\overline{\theta}}_N} = \overline{\overline{A}}\,\overline{\overline{J}}^{-1}\overline{\overline{A}}^T \tag{4.11}$$

where the transformation matrix $\overline{\overline{A}}$ is given by

$$\overline{\overline{A}} = \begin{bmatrix} \frac{\partial f_1}{\partial \Phi_1} & \frac{\partial f_1}{\partial \Phi_2} & . & . & . & . & . & \frac{\partial f_1}{\partial \Phi_{2n}} \\ \frac{\partial \zeta_1}{\partial \Phi_1} & \frac{\partial \zeta_1}{\partial \Phi_2} & . & . & . & . & . & \frac{\partial \zeta_1}{\partial \Phi_{2n}} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & \frac{\partial f_n}{\partial \Phi_{2n}} \\ . & . & . & . & . & . & . & \frac{\partial \zeta_n}{\partial \Phi_{2n}} \end{bmatrix} \tag{4.12}$$

$\hat{\overline{\theta}}_N$ is an estimator of the parameter vector $\overline{\theta} = [f_1, \zeta_1, f_2, \zeta_2, ..., f_n, \zeta_n]^T$. The above estimation of $\overline{\overline{A}}$ will only be accurate if the function is sufficiently smooth since it corresponds to a linear approximation of the function describing the inverse transformation from AR- parameters to the parameters $\overline{\theta}$, see e.g. Kirkegaard [48] and Jensen et al. [49].

5. EXPERIMENTAL RESULTS AND DISCUSSION

In this chapter the results from the experimental investigations are presented and discussed. In order to select a data acquisition strategy and to investigate the assumptions for using an ARMA-model some preliminary investigations were performed. The results of these investigations are described in section 5.1. It is explained how the recorded signals were sampled and signal processed before system identification was performed. Further, it is described how the ARMA-model was selected and validated. In section 5.2 the estimated modal parameters, natural frequencies and modal damping ratios, are given and discussed for the undamaged mast as well as the damaged mast.

5.1 Testing and General Results

In the period from December 92 to June 93 twenty measurements sessions were performed with the undamaged mast . The dates of the sessions were selected such a data base containing measured responses due to different wind-directions and wind-speeds were created. At a measurement session 10 time series were recorded for each transducer, i.e. accelerometers as well as cup-anemometer and wind-vane. In the same period 2 measurement sessions were performed where damages were simulated at the mast. In enclosure E a list of the measurements sessions is given. The list contains date, damage state and air-temperature. The list shows that the lowest temperature in the period was -5 ^{o}C and the highest 20 ^{o}C.

5.1.1 Identification and Decomposition of Rotational Modes

In this section rotational modes are identified from bending modes. Figure 5.1 shows the autospectra, the transfer function and the coherence function, respectively, of the signals obtained from the accelerometers at the location 1.1 and 1.3. The 95% confidence intervals are displayed on the autospectra with dashed lines. Since the mast is symmetrical the accelerometers 1.1 and 1.3 will measure responses nearly equal in magnitude for both bending and rotational vibrations and have the same sign in bending vibration but opposite sign in rotational vibration. Adding signals for two symmetrical locations doubles the bending response and deletes the rotational response. Subtracting the signals doubles the rotational response and delete the bending response. Figure 5.1 shows that the mast has some clearly seen natural frequencies below 30 Hz. Further, it is seen that the coherence function indicates some rotational frequencies at approximately 8, 25 and 45 Hz, respectively. Figure 5.2 shows the autospectra of the preprocessed signals obtained from the accelerometers at the location 1.1 and 1.3.

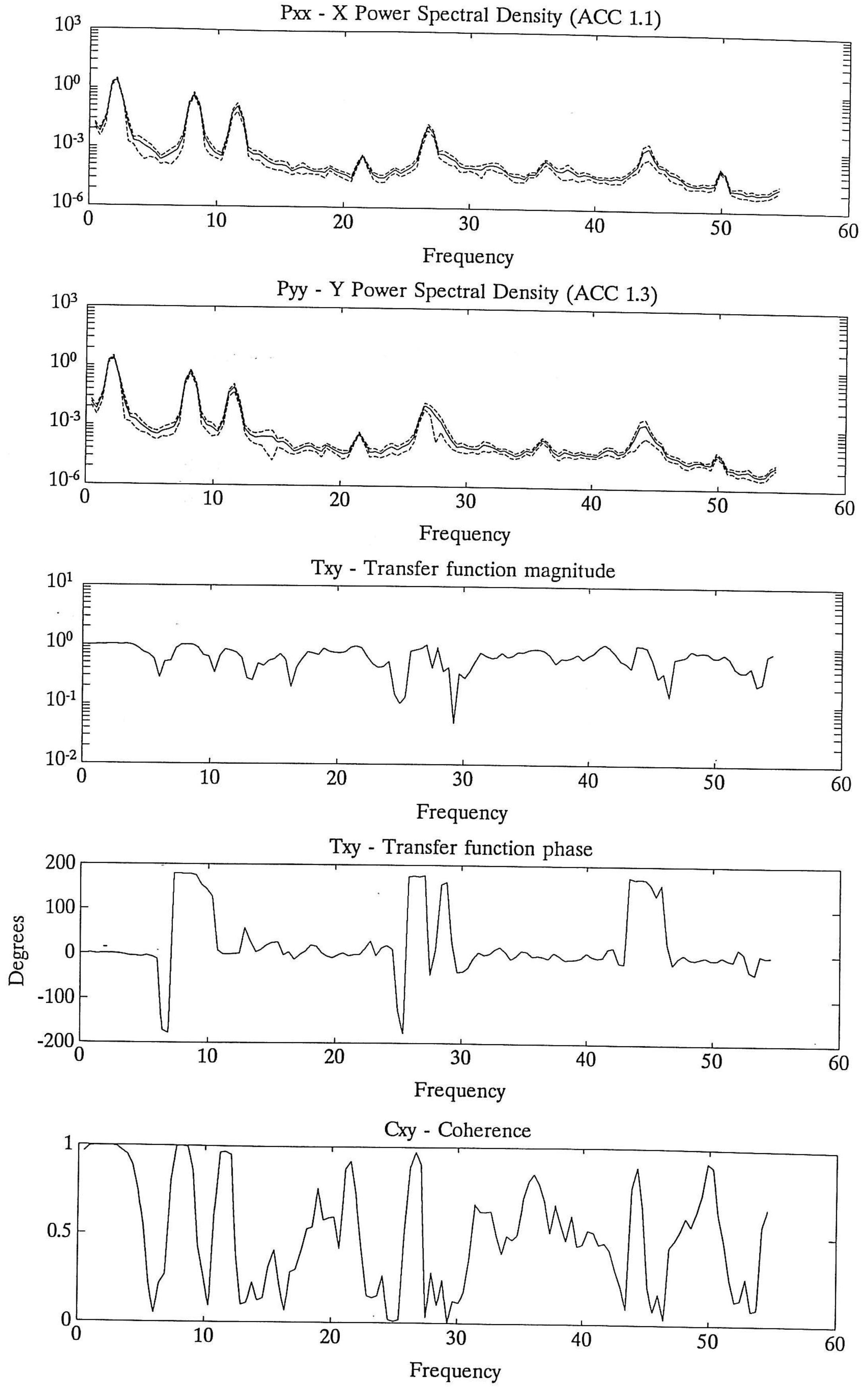

Figure 5.1: Autospectra, transfer function and the coherence function, respectively, of the signals from accelerometer at location 1.1 and 1.3.

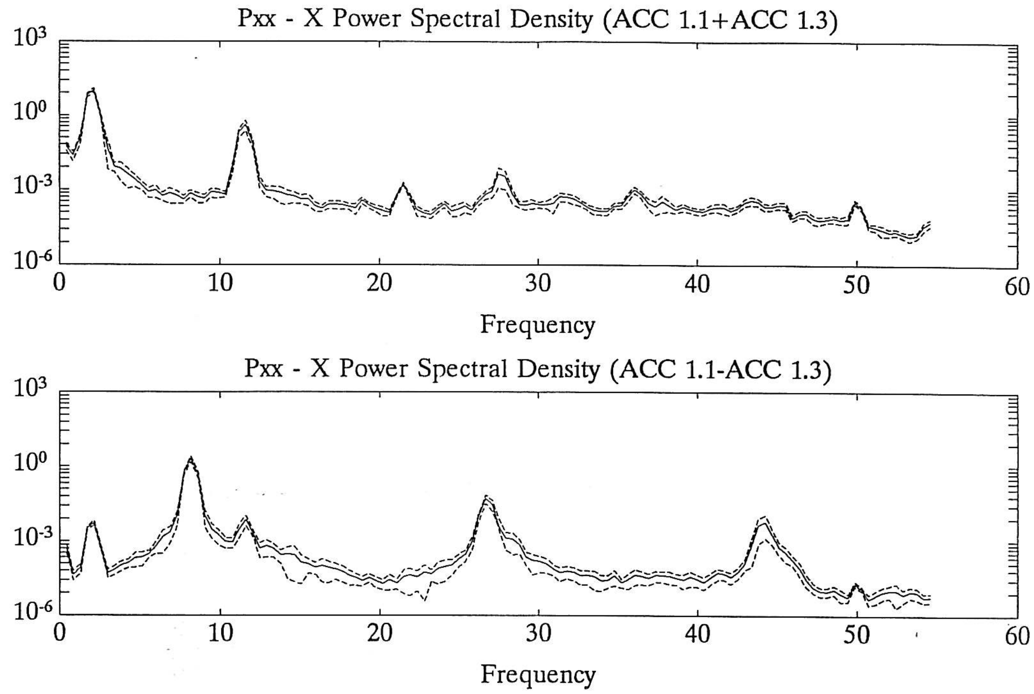

Figure 5.2: Autospectra of the preprocessed signals from accelerometers at location 1.1 and 1.3, measurement 1.a.

Figure 5.2 clearly shows the effect of decomposition of the natural peaks. The figure shows that the mast has natural bending frequencies at approximately 2, 11 and 28 Hz, respectively. Further, it is seen that natural rotational frequencies exist at approximately 8 and 26 Hz. It is seen that the decomposition into the rotational modes and bending modes corresponds to the FE-results in section 3.2.

It may be noticed that the system identification in the following is not performed based on signals which have been preprocessed in order to make a decomposition of modes. If the frequencies of the bending and rotational vibrations had been close to each other a decomposition in the data analysis would have been needed.

5.1.2 Data Acquisition Strategy

Recordings in their initial (i.e., raw) form are generally not appropriate for system identification. The precision of the identification can be improved significantly by processing the data prior to identification. The processing involves, in general, the following steps: 1) removal of mean and outliers; 2) filtering; 3) selection of sampling frequency and decimation; 4) synchronisation. Non-zero mean values in records represent the static components and very low frequency drifts. Since we are interested in identifying dynamic properties, these components should be removed so that they do not distort the identification. The removal can be done either by simply subtracting the arithmetic mean of the data, or by using high-pass filters. Outliers in records are isolated erroneous large peaks that may occur due to various reasons, such as temporary sensor failures, or accidental knocks on the instruments. These peaks can adversely influence the identification results, especially when the identification is based on the least-squares criterion. The data should be visually checked for such outliers prior to identification. Filtering aims to remove the frequency components of the signal that are dominated by noise and prevents antialiasing. Naturally, any model components existing in those frequency regions, eliminated by filtering, would not be identified. Decimation means to decrease the sampling rate of the records. A given record would contain information up to the Nyquist frequency that is equal to half the sampling frequency. If the expected highest frequency in the structure is much smaller than the Nyquist frequency, the sampling rate can be decreased, i.e. decimate the record, without loosing any information. The rate of decimation, i.e. the number of data points that will periodically be skipped, is determined in such a way that the new Nyquist frequency is only slightly higher than the expected highest frequency. Before the decimation the record has to be low-pass filtered beyond the Nyquist frequency. The advantages of the decimation are the reduced noise effects, since noise generally has a broader spectral band, and the reduced data size. The last step in data processing is the synchronisation of the input and output, if they are not recorded synchronously. Synchronisation can also be handled during the identification by selecting the time-delay parameter appropriately. It is recommended to synchronise the records prior to identification, so that any time delay determined during the identification represents the delay in the structural response rather than the unsynchronized recording.

In the light of the above quotation, it is now explained how the recorded signals have been signal processed. From the preliminary spectra, figure 5.1-5.2 it was decided to concentrate the identification on the first two bending modes parallel to the x-axis and y-axis, respectively, and the first rotational mode. This means that it was the signals from the accelerometer 1.1, 1.2, the cup-anemometer and the wind vane which were analyzed. From accelerometer 1.1 signals the first and second bending natural frequency parallel to x-axis, and the first rotational natural frequency can be estimated. In the same way, the first and the second bending natural frequency parallel to the y-axis and the first rotational natural frequency can be estimated from accelerometer 1.2 signals. Prior to the identification the acceleration signals were detrended and removed from outliers. Further, the signals from the accelerometers were low-pass filtered with a cut-off frequency selected to 13.3 Hz corresponding to 70% of the Nyquist frequency. The signals from the cup-anemometer and the wind-vane were not filtered. The sampling frequency was selected from results given in Hummelshøj et al. [50] where it is shown by a simulation study that a sampling frequency equal to approximately 38 Hz will give the best reduction of bias of the modal parameter estimates. Further, by using this sampling frequency, it is shown that only a limited reduction of the variance of the modal parameter estimates can be obtained by using more than 8000 number of data points N. This implies that 8000 points were sampled by 38 Hz from accelerations signals and signals from the cup-anemometer and the wind-vane. The signals were not high-pass filtered in order to remove low-frequency drifts in the data.

In enclosure F a table is given with the standard deviation of the acceleration signals obtained with accelerometer 1.1 and 1.2. Further, the direction and the speed of the wind, respectively, are given. These quantities are given for each measurement. Examples of a time series and a spectrum from different measurement sessions are shown in enclosure G. At the plot of the time series the standard deviation (std) of the acceleration signal is shown.

5.1.3 Check of Assumptions

In section 4.2.1 it is stated that the measured data have to be stationary Gaussian distributed if an ARMA-model based system identification should be performed. Further, the structure should have linear response. The validity of these assumptions should be investigated prior to the system identification.

The stationarity of the response is assumed based on experiences from previous response measurements on wind excited structures.

The linearity and normality have been investigated by estimating the probability distribution function of the response as well for the undamaged structure as the undamaged structure. By comparing this estimated distribution function with the theoretical given normal distribution function a qualitative evaluation of the normality can be performed. If the structure is linear the resulting response will be Gaussian due to the Gaussian excitation. In figure 5.3 estimated probability functions for different wind loads are shown. The measured response have been normalized with respect to the standard deviation of the acceleration signal. A picture is used where a normal probability function will be a straight line.

a) b)

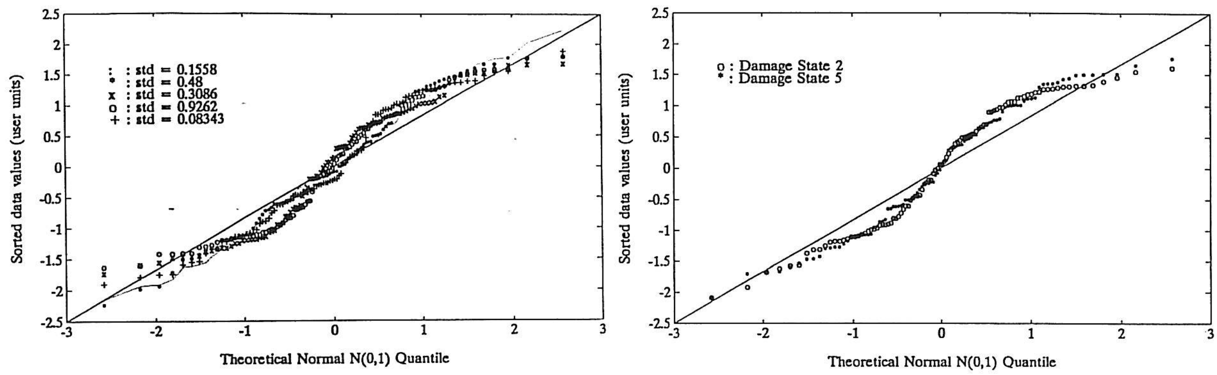

Figure 5.3: Check of Normality. a) Undamaged structure. b) Damaged structure.

It is seen from figure 5.3 that the measurements can be assumed approximately Gaussian, both for the undamaged as well for the damaged structure.

5.1.4 Selection and Validation of ARMA-model

In the following it is explained how the ARMA-model was selected and validated. The results are given for a recorded signal from measurement number 1.1 and and accelerometer 1.1, see enclosure F. The modal parameters were estimated by an ARMA-model as described in section 4.2.1.

The model-order was estimated by plotting values of the AIC criterion for model order selection given in section 4.2.3 as function of model order, see figure 5.4. From figure 5.4 it was determined that an 6th-order model should be used for the identification.

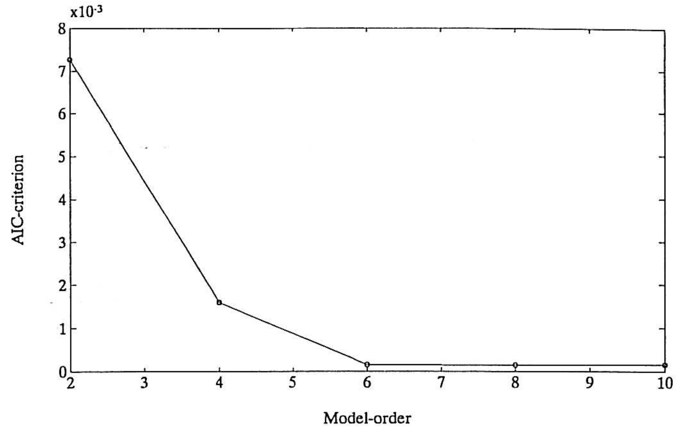

Figure 5.4: AIC as function of model order.

Figure 5.5 shows a plot of the poles (x) and zeros (o) and it is seen that all the poles and zeros are inside the unit circle in the complex plane. The poles and zeros are given with confidence regions corresponding to three standard deviations. If these regions overlap, a lower model order should have been tried, since this is a result of a near pole-zero cancellation in the dynamic model indicating that the model order is too high. The most dominant mode of the system is the one corresponding to the pole closest to the unit circle.

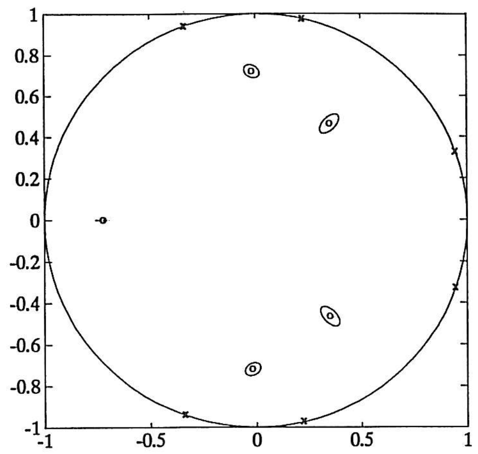

Figure 5.5: Pole-Zero plot.

As discussed in section 4.2.2, after the model is selected and the parameters are determined, the next step is to check the validity of the model. The match of the power spectrum obtained by a Fast Fourier Transformation and the spectrum obtained from the ARMA-model are shown in figure 5.6. The figure shows a good match.

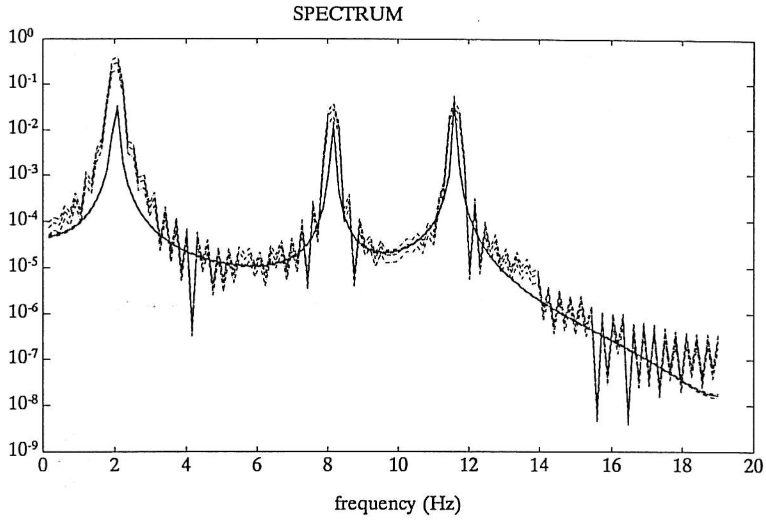

Figure 5.6: Comparison of direct estimated spectrum and spectrum obtained from the ARMA-model (full-line).

Next the residuals of the identification are checked. Residuals are defined as the difference between the model output and the recorded output signal. In order to have a valid identification, the residuals should be a white-noise sequence. The plot of the first 2000 residuals and its spectrum are given in figure 5.7.

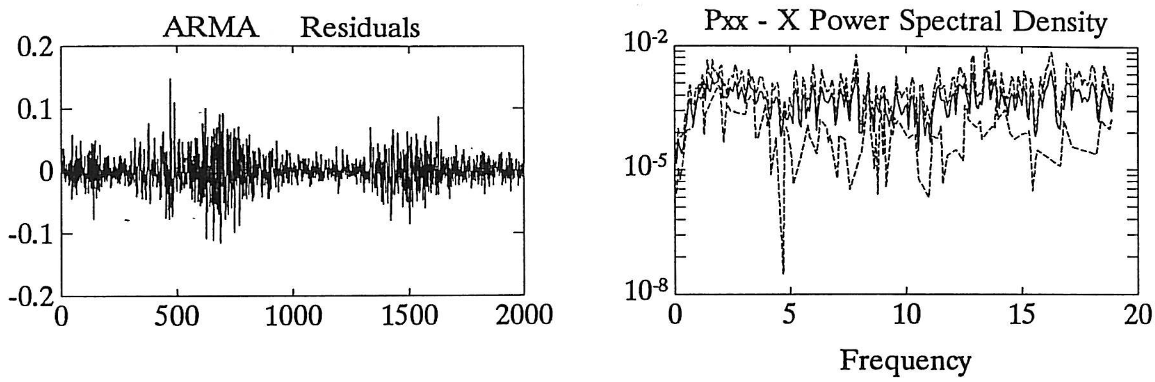

Figure 5.7: Residual time series and spectrum of the residual time series.

Visual inspection of the spectrum in figure 5.7 suggests that the residuals are close to a white-noise sequence, since the peaks are distributed in all frequencies. A more accurate check is to test the autocorrelation of the residuals, see figure 5.8.

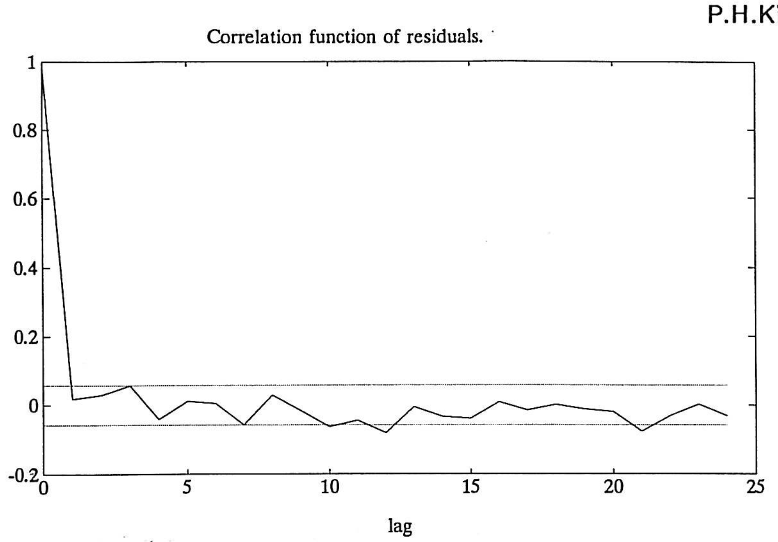

Figure 5.8: Autocorrelation of the residuals time series.

Two straight lines in the figure show the 99 % confidence level. For model validity, i.e. whiteness of residuals, the autocorrelation should not exceed these levels, except at zero lag. Figure 5.8 shows that the autocorrelation remains, for the most part, within the limits, and therefore validate the model. As a final test for model validity, a comparison of model output with recorded output. This is a more strict test than the previous ones. However, figure 5.9 shows that the match is fairly good. Based on all the above checks, it can be concluded that the estimated ARMA-model for the mast is satisfactory.

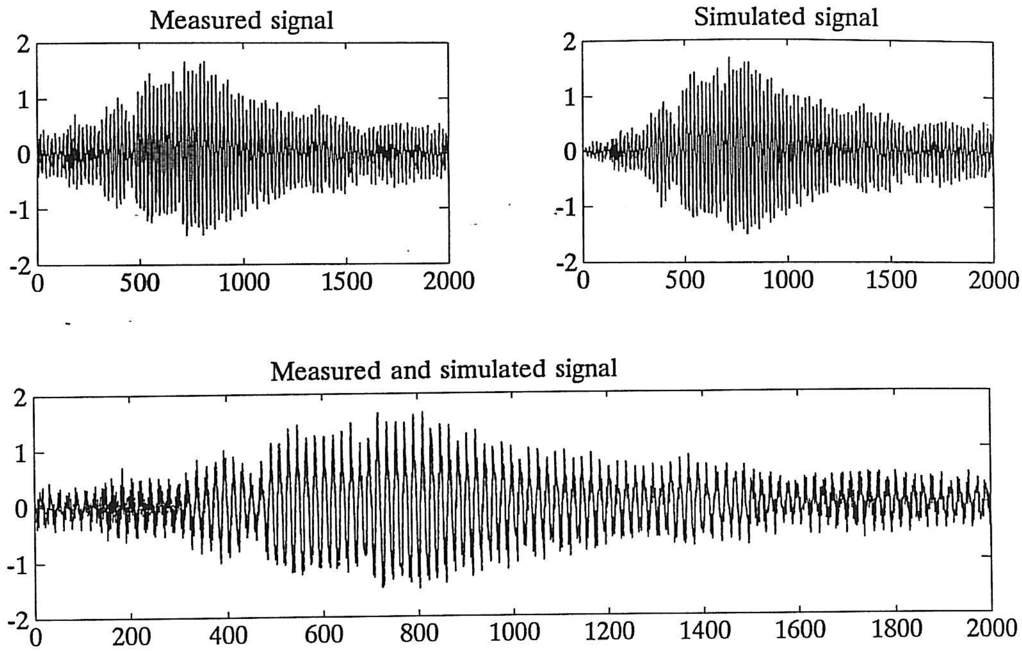

Figure 5.9: Comparison of calculated accelerations with recorded accelerations, plotted separately and together.

5.2 Modal Parameters

In this section the estimated modal parameters, natural frequencies and modal damping ratios are presented and discussed. The modal parameters were estimated by using the MATLAB, see PC-MATLAB [51], based program STDI, see Kirkegaard et al. [7]. A typical output from STDI is shown in enclosure H.

5.2.1 Undamaged Mast

In enclosure I the estimated natural frequencies and modal damping ratios of the undamaged mast have been given with their standard deviation. As mentioned above it is the natural bending frequencies no. 1 and no. 4, the natural bending frequencies no. 2 and no. 5 and the natural frequency no. 3 corresponding to deflection parallel to the x-axis and defection parallel to the y-axis and rotation, respectively.

The estimates of the natural frequencies f_i and the modal damping ratios ζ_i are shown as functions of measurement number in figure 5.10. The 20 estimates in each figure have been obtained by combining the measured estimates of natural frequencies and modal damping ratios, respectively, from a measurement session. At each measurement session 10 times series were recorded, implying 10 estimates of the natural frequencies and modal damping ratios, respectively. These 10 estimates from one session have been combined in the following way

$$f_i = \frac{\sum_{k=1}^{N_p} \frac{f_{i,k}}{\sigma^2_{f_{i,k}}}}{\sum_{k=1}^{N_p} \sigma^{-2}_{f_{i,k}}} \tag{5.1}$$

$$\zeta_i = \frac{\sum_{k=1}^{N_p} \frac{\zeta_{i,k}}{\sigma^2_{\zeta_{i,k}}}}{\sum_{k=1}^{N_p} \sigma^{-2}_{\zeta_{i,k}}} \tag{5.2}$$

where $\sigma^2_{f_{i,k}}$ and $\sigma^2_{\zeta_{i,k}}$ are the variance of the the estimates of natural frequencies $f_{i,k}$ and modal damping ratios $\zeta_{i,k}$, respectively, from the $k'te$ of the N_p measurement sessions.

The solid lines in figure 5.10 indicate a mean value of the 20 estimates while the dashed lines give an interval between the mean value plus one per cent and the mean value minus one per cent for the natural frequencies. In the same way an interval corresponding to the mean value plus ten per cent and the mean value minus ten per cent is shown with dashed line for the modal damping ratios.

Figure 5.10 shows that the measured natural frequencies vary approximately only few per cent while the modal damping ratios vary more than twenty per cent. It is seen that the bending natural frequencies are more sensitive than the rotational frequency. The standard deviation of the natural frequencies and the modal damping ratios, see enclosure I, are approximately 0.003 Hz and 0.001, respectively. This indicates that the variation of the measured modal parameters is due to changes in the environmental conditions and not only due to randomness.

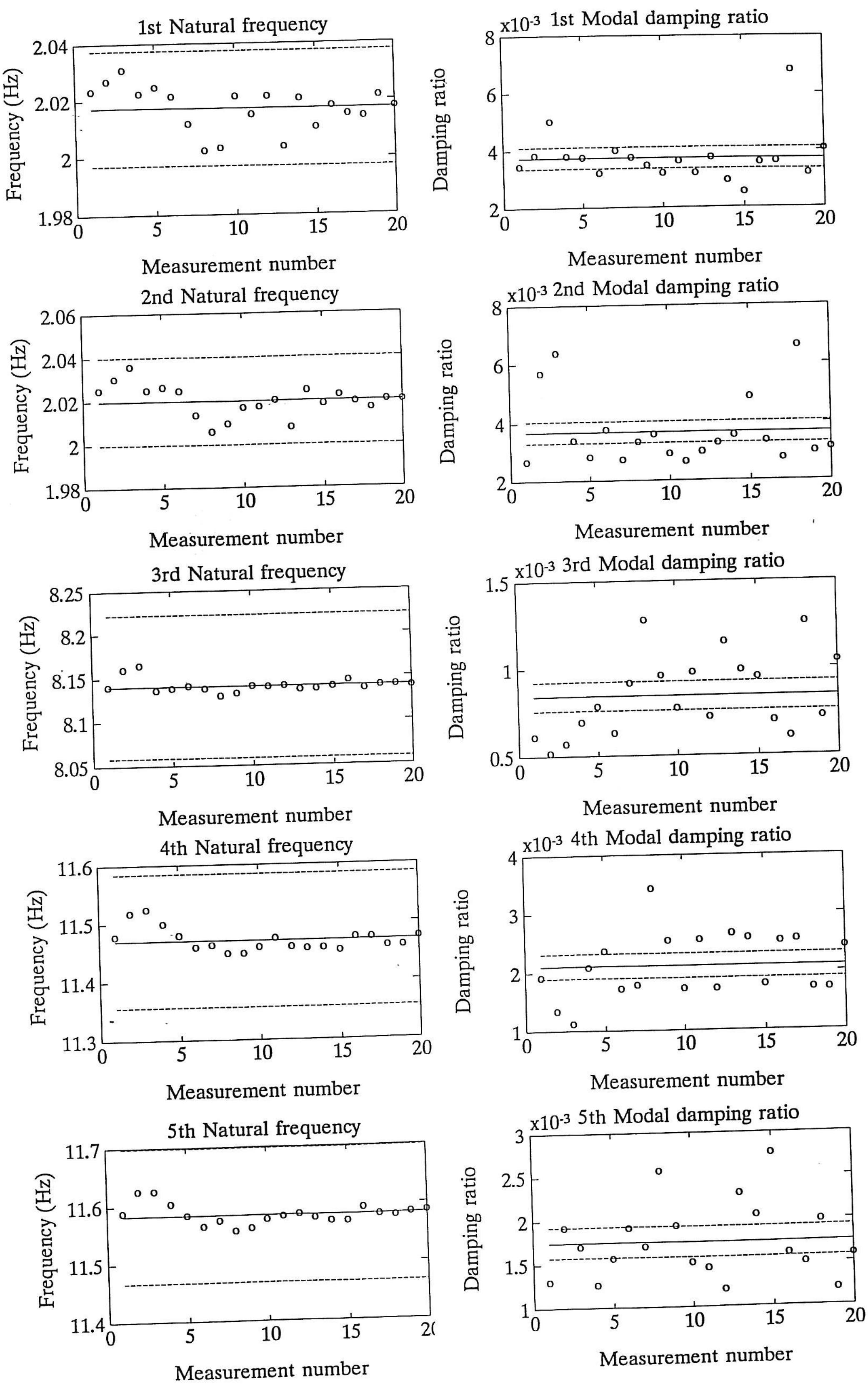

Figure 5.10: Estimated natural frequencies and modal damping ratios as function of measurement number. (Solid line show the mean value and dashed lines show the mean value plus/minus one percent).

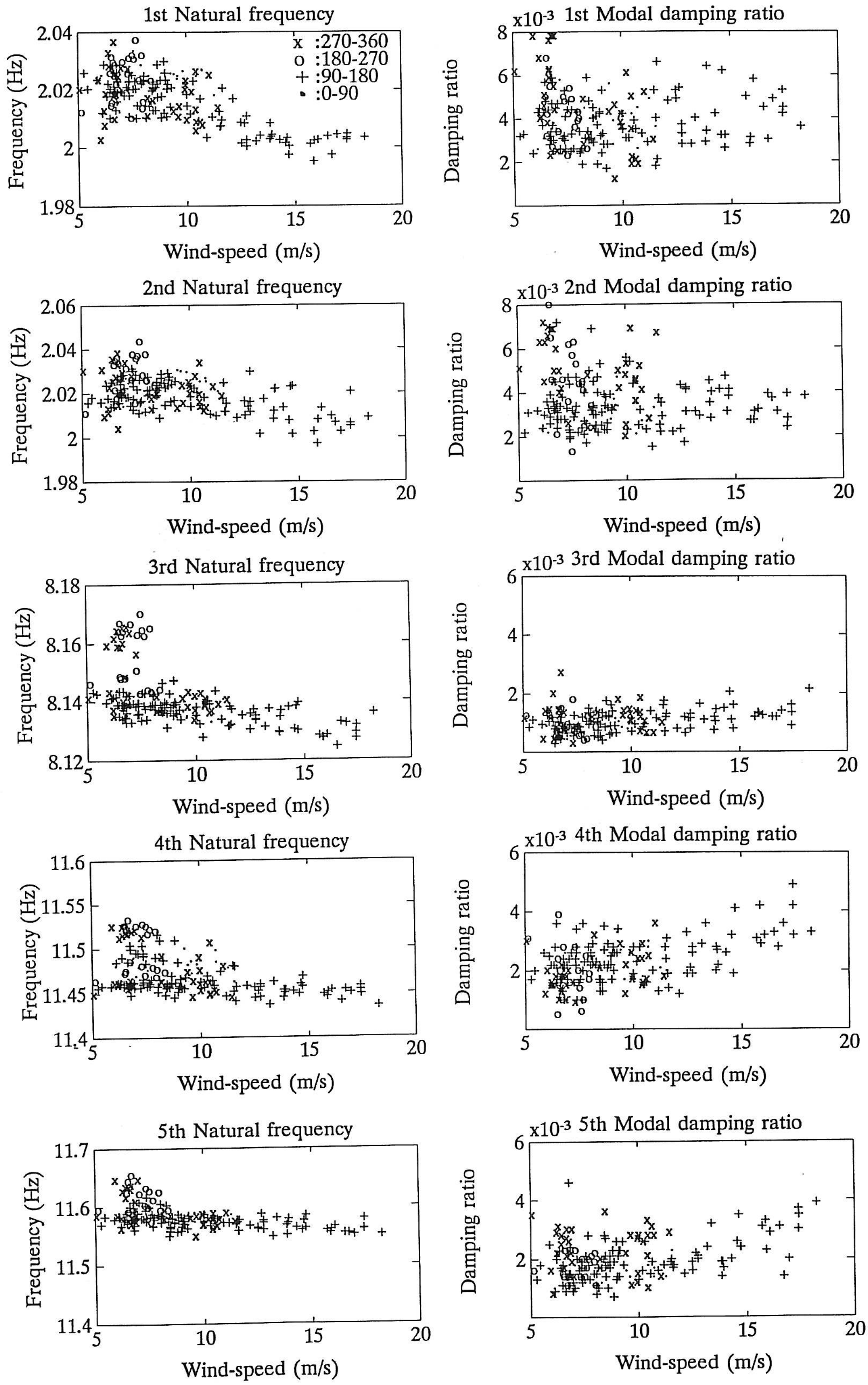

Figure 5.11: Natural frequencies and modal damping ratios as function of wind-speed and wind-direction.

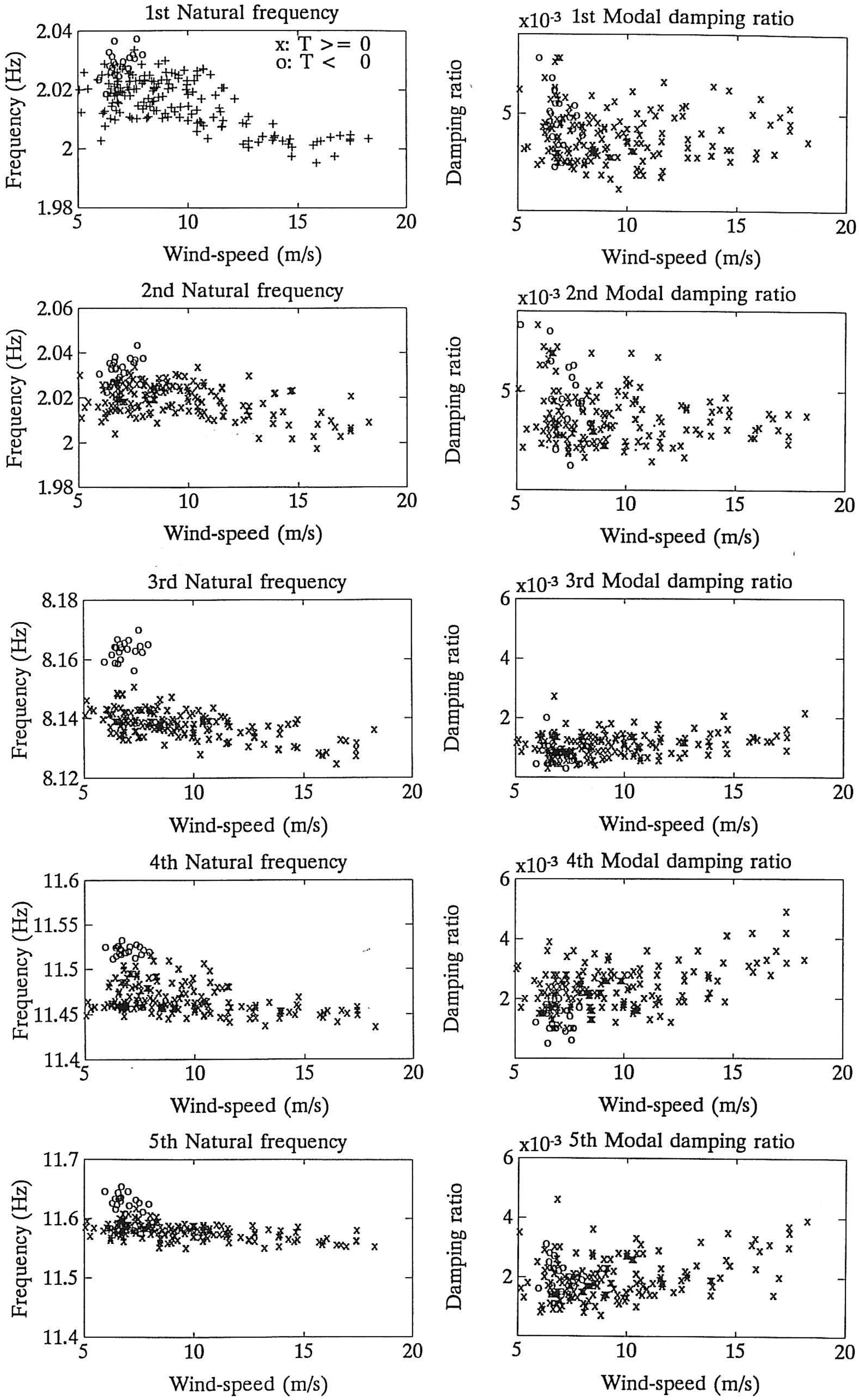

Figure 5.12: Natural frequencies and modal damping ratios as function of wind-speed and air temperature.

In order to investigate the sensitivity of modal parameters with respect to the wind-direction and the wind-speed the 200 estimates of the natural frequencies and modal damping ratios, respectively, are shown in figure 5.11 as functions of the wind-speed. The estimates have been divided into 4 groups. Each group corresponds to a wind-direction interval of 90 degrees.

Figure 5.11 shows that the modal parameters are sensitive to the wind-speed. However, it is most clear for the first and second natural frequency. Further, it is seen that the natural frequencies have an increase for a wind-speed corresponding to 7-8 m/s when the wind-direction is changed. However, this change can also be a consequence of a change in temperature, see figure 5.12. In figure 5.12 the 200 estimates of the natural frequencies and modal damping ratios, respectively, are shown as functions of the wind-speed. The estimates have been divided into 2 groups. One group corresponds to the estimates obtained from the measurements where the air temperature was lower than 0 ^oC and higher than 0 ^oC, respectively. It is seen that the increase in natural frequencies for a wind-speed corresponding to 7-8 m/s can be due to an air temperature below 0 ^oC and not necessarily a change in the wind-direction. However, more data must be obtained in order to investigate this problem.

5.2.2 Damaged Mast

At two different measurement sessions, measurement 4 and 6, the natural frequencies and modal damping ratios were estimated for seven different damage states. In section 2.2 it is explained that the eight lower diagonals in the mast were cut and provided with joints consisting of 4 splice plates. By removing some of these splice plates the seven damage states (1,2,5,6,9,10,11) were simulated. The damage state 1,2,5 and 6 correspond to the damage state defined in section 3.1 while damage states 9 and 11 correspond to fifty per cent reduction of the sectional area of element AB101 and AB102, respectively. Damage state 10 corresponds to fifty per cent reduction of the sectional area of element AB101, BC101, CD101 and DA101. The measured natural frequencies and the modal damping ratios have been given in enclosure J. In figure 5.13 the measured natural frequencies and modal damping ratios are shown as functions of damage state. The solid lines indicate the mean value from figure 5.10. Figure 5.13 shows that the modal parameters are sensitive to a damage corresponding to removing one of the lower diagonals, i.e. damage states 1,2,5 and 6. It is seen that the change in the bending natural frequencies depends on the location of the damage. Figure 5.13 also shows that the rotational frequency is more sensitive to a damage than the bending frequencies. Further, the modal parameters seem to be insensitive to damages corresponding to damage states 9,10 and 11. However, in section 5.2.1 it is shown that the modal parameters are sensitive to environmental conditions. Therefore, in order to distinguish between a change in the modal parameters due to a damage or the environmental conditions, modal parameters corresponding to the same environmental conditions have to be compared. In figure 5.14a and 5.14b the measured natural frequencies and modal damping ratios from measurement 4 and 6 are shown as functions of damage state, respectively. The solid line in figure 5.14a shows the lower bound of the 95% confidence level of the natural frequencies from measurement 3. The estimates are assumed Gaussian distributed. In the same way in figure 5.14b the lower bound of the 95% confidence level of the natural frequencies from measurement 5 is shown. The measurements 3 and 5 (undamaged) correspond to measurements 4 and 6 (damaged), respectively, with respect to environmental conditions, i.e. approximately the same wind-speed, the wind-direction and the air-temperature. This means that a change in the measured natural frequencies can be interpreted as a change due to a damage and not to a change in the environmental conditions. Figure 5.14 shows that it is possible to detect a damage in the mast corresponding to removal of one of the lower diagonals, damage states 1,2,5 and 6. Further, a damage, damage states 9 and 11, corresponding to a fifty per cent reduction of the sectional area can be detected. However, if such a damage should be detected it is important to compare modal parameters from the damaged and undamaged mast, respectively, obtained under the same environmental conditions.

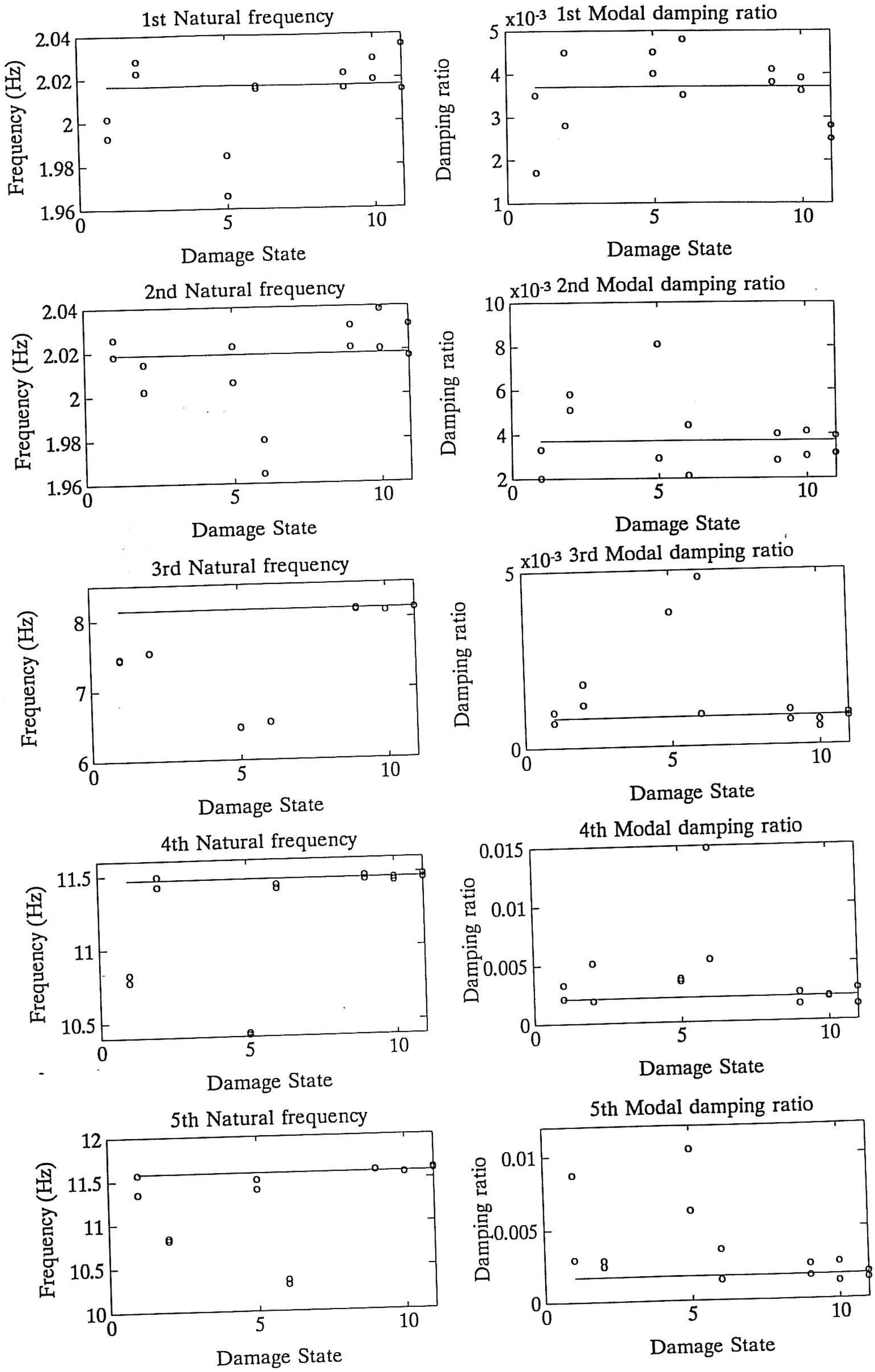

Figure 5.13: Estimated natural frequencies and modal damping ratios as function of damage state. (Solid line is the mean value from figure 5.10)

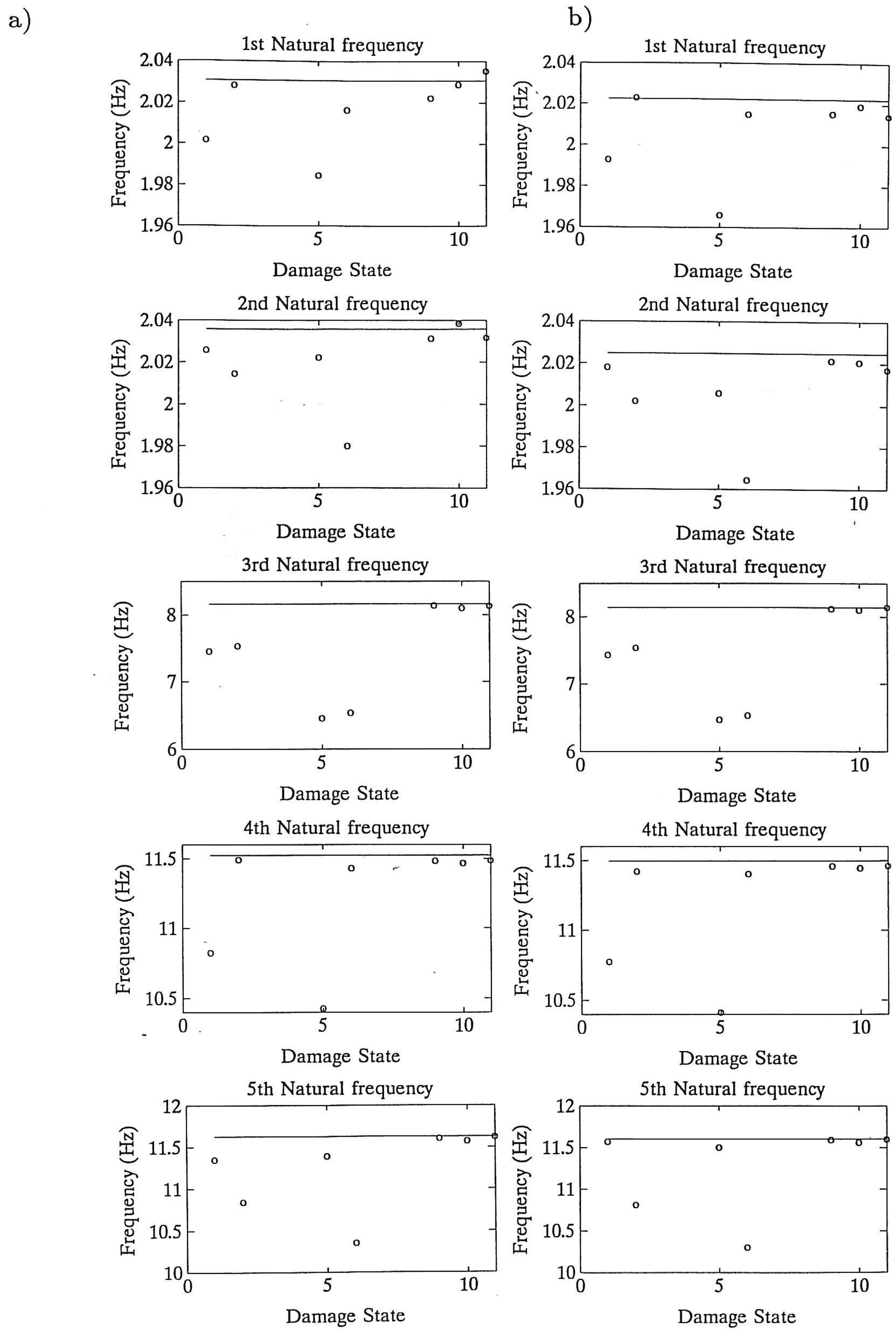

Figure 5.14: Estimated natural frequencies from measurement 4 (a) and 6 (b) as function of damage state. (Solid lines are the lower bound of the 95% confidence level for estimated natural frequencies from measurement 3 (a) and 5 (b), respectively)

NOMENCLATURE

α	: wind-direction
$\overline{\overline{M}}$	: mass matrix
$\overline{\overline{K}}$	: stiffness matrix
$\overline{\overline{C}}$	: damping matrix
$\overline{y}(t)$	: displacement vector
$\overline{u}(t)$	: force vector
n	: degrees-of-freedom
Y_t	: discrete stochastic response process
y_t	: realization of Y_t
Φ_i	: Auto Regressive parameters
$\mathcal{O}_1$	: Moving Average parameters
$\mathcal{E}_t$	: noise process
e_t	: realization of $\mathcal{E}_t$
N	: number of sample points
V_N	: variance of e_t
$\epsilon_t(\overline{\Phi})$	: prediction error process
y_t^M	: measured response
$\hat{y}_t$	: estimated response
λ_i	: root of characteristics polynomial
ζ_n	: damping ratio of n'th mode
ω_n	: natural cyclic frequency n'th mode
Δt	: sampling interval
FPE	: Final Prediction Error
AIC	: Information Theoretic Criterion
$\overline{\overline{J}}$	: Fisher information matrix
$\lambda_{\mathcal{E}}$	: variance of noise process
$E[\cdot]$	: expectation operator
$\overline{\Phi}$	: vector including AR-parameters
$\overline{\Psi}(t,\overline{\Phi})$	: stochastic process
$\overline{\psi}(t,\overline{\Phi})$	: realization of $\overline{\Psi}(t,\overline{\Phi})$
$\overline{\overline{A}}$	: transformation matrix given by (4.12)
$\overline{\theta}$	: modal parameter vector
$\hat{\overline{\theta}}_N$	: estimate of $\overline{\theta}_N$
$\overline{\overline{C}}_{\hat{\overline{\theta}}_N}$	: covariance matrix

f_n : natural frequency of mode n
$\sigma^2_{f_{i,k}}$: the variance of the the estimates of natural frequencies $f_{i,k}$
$\sigma^2_{\zeta_{i,k}}$: the variance of the modal damping ratios $\zeta_{i,k}$
$f_{i,k}$: the natural frequencies from the $k'te$ of the N_p measurement sessions
$\zeta_{i,k}$: the modal damping ratios $\zeta_{i,k}$ from the $k'te$ of the N_p measurement sessions
N_p : number of measurement sessions.

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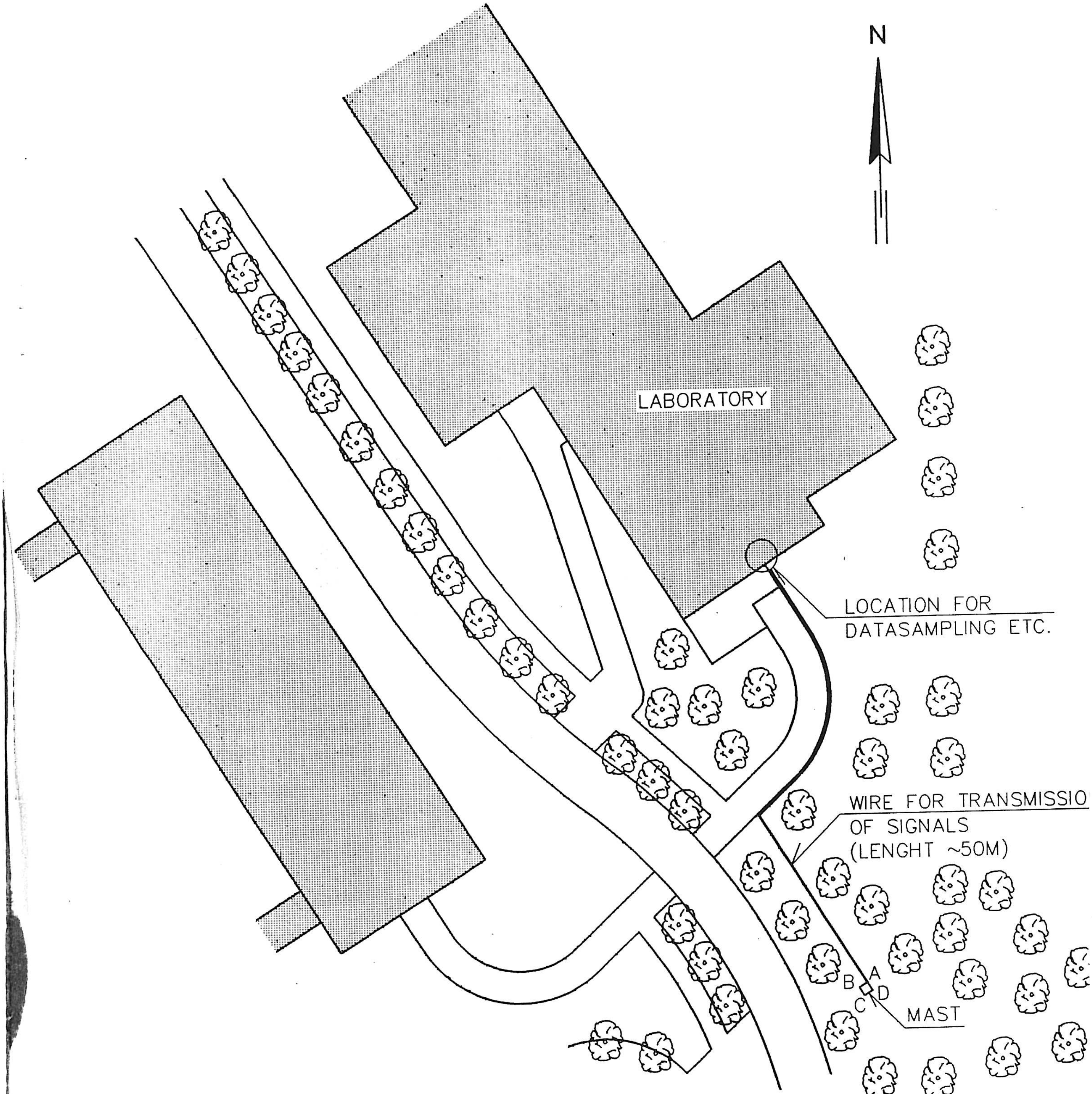
ENCLOSURE A

Site Plan and Elevation of Mast



RH&H CONSULT
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STVF, PILOT PROJECT

SITE PLAN

Job No.: 92.4021

Scale: 1:500 Dwg.No.: 1

Drawn by: ARA/HW Controlled: Approved: Date: File: 1

Kjaerulfsgade 2
DK-9400 Nörresundby
Denmark

Phone +45 98 17 38 00
Telefax +45 98 17 57 33
Telex 37 108 ramhan dk

–a member of the RH&H Consultancy Group

NOTE:

UDENOTED MEASUREMENT IN MM.

⟶ MEAUREMENT DIRECTION FOR ACCELEROMETER

RH&H CONSULT
RAMBØLL HANNEMANN & HØJLUND A/S

STVF, PILOT PROJECT		Job No.: 92.4021	
MAST		Scale: 1:100	Dwg.No.: - 2
Drawn by: ARA/HW Controlled:	Approved:	Date:	File: 2

Kjaerulfsgade 2
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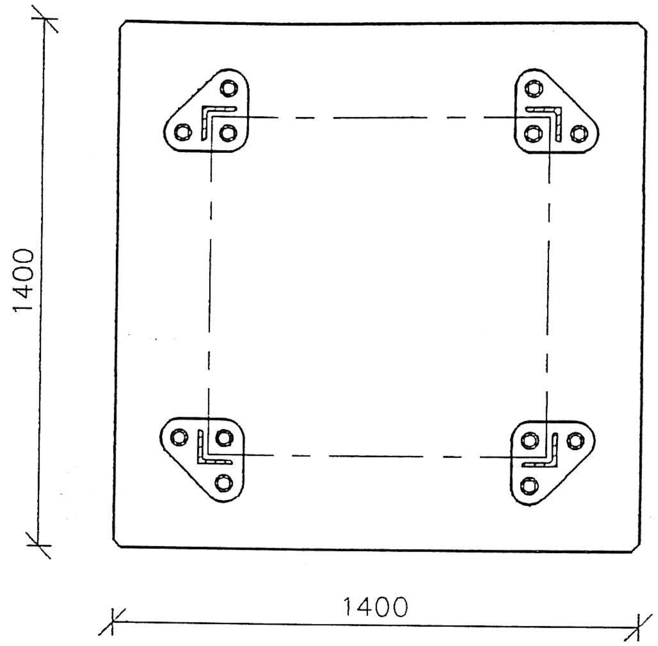
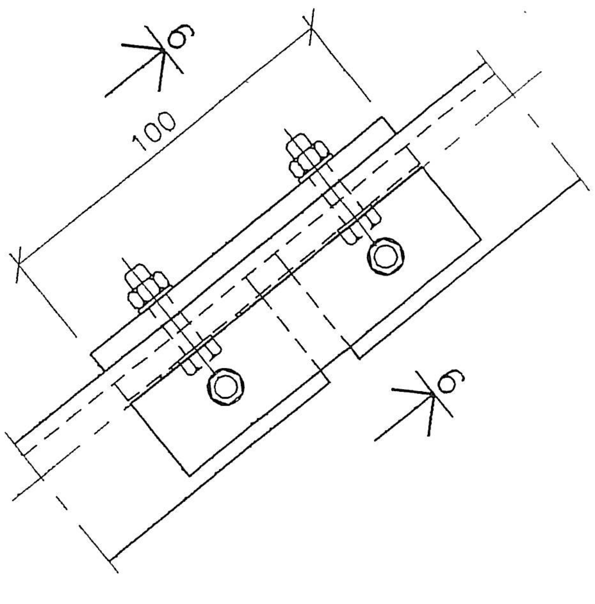
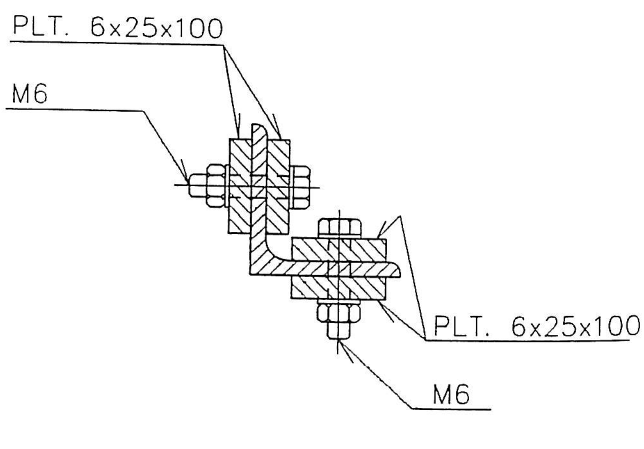


SECTION 5–5, 1:20



DETAIL 1, 1:2



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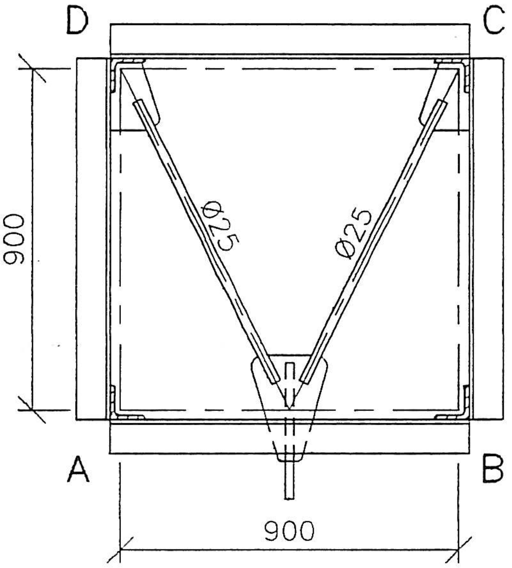
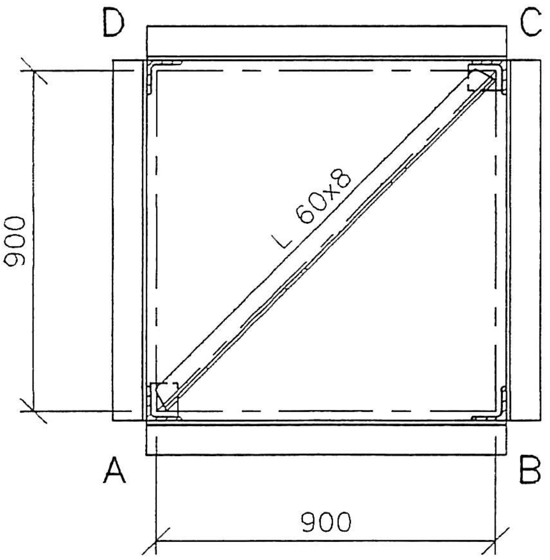
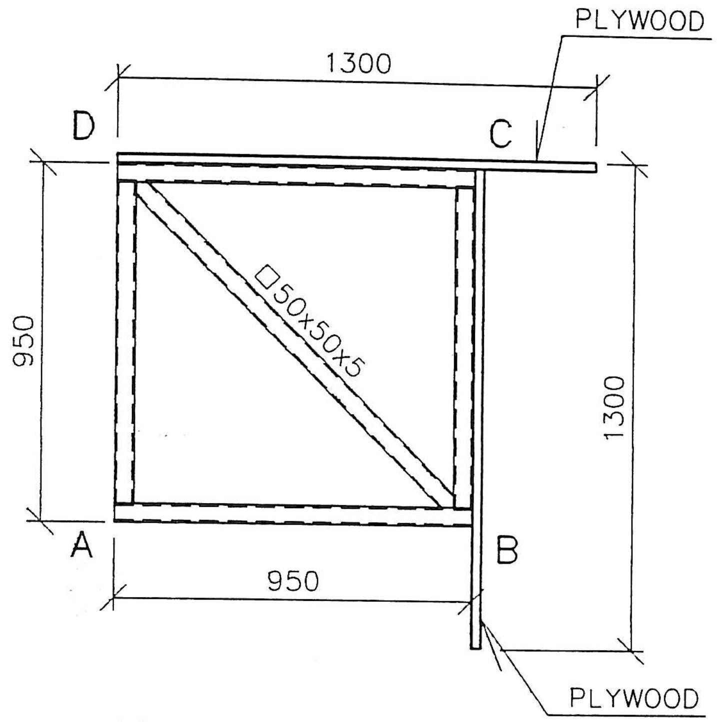


SECTION 3–3, 1:20

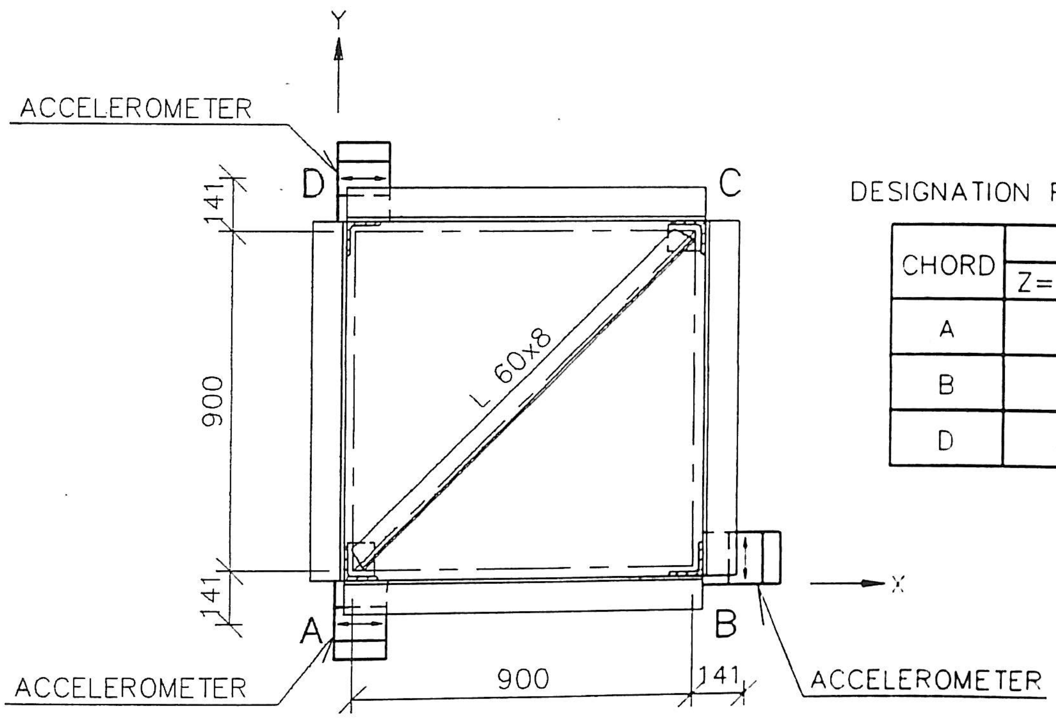
ER



SECTION 4–4, 1:20



SECTION 1–1, 1:20



SECTION 2–2, 1:20

DESIGNATION FOR ACCELEROMETER

CHORD	LEVEL	
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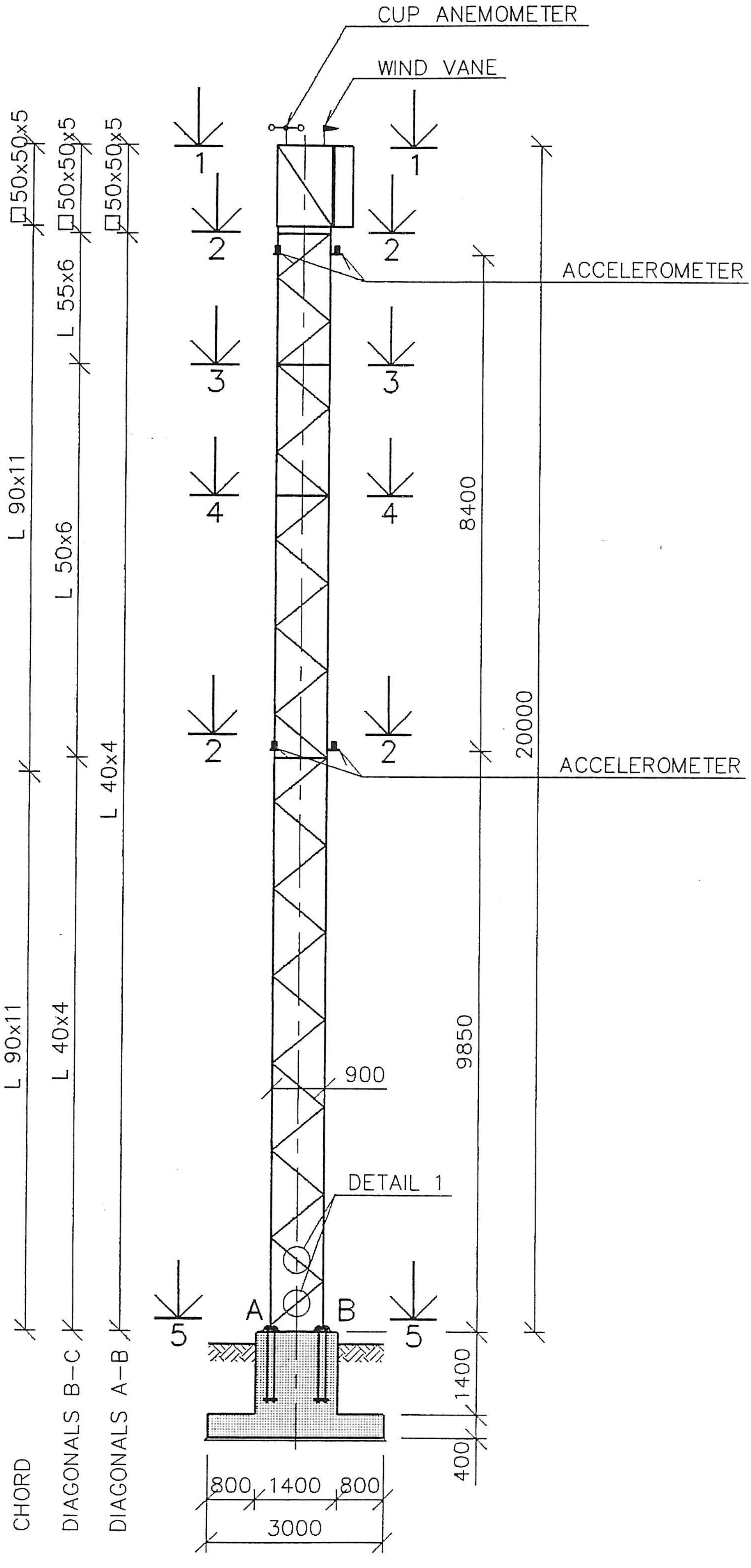
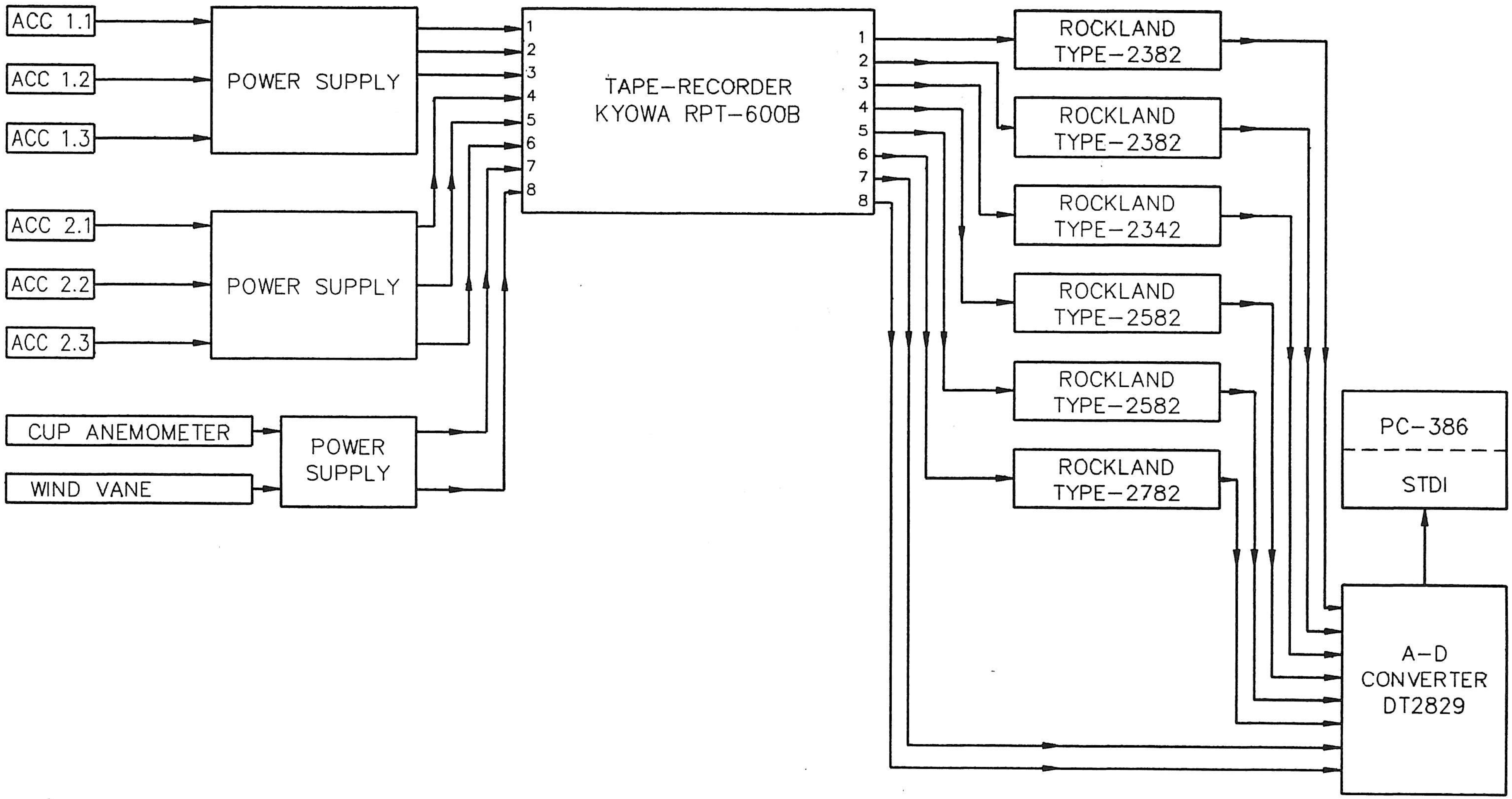
A	2.1	1.1
B	2.2	1.2
D	2.3	1.3



CUP ANEMOMETER
WIND VANE
ACCELEROMETER
ACCELEROMETER
DETAIL 1
□50x50x5
□50x50x5
□50x50x5
L 55x6
L 90x11
L 50x6
L 40x4
L 90x11
L 40x4
CHORD
DIAGONALS B–C
DIAGONALS A–B
1
1
2
2
3
3
4
4
2
2
5
5
A
B
8400
20000
9850
900
1400
400
800
1400
800
3000

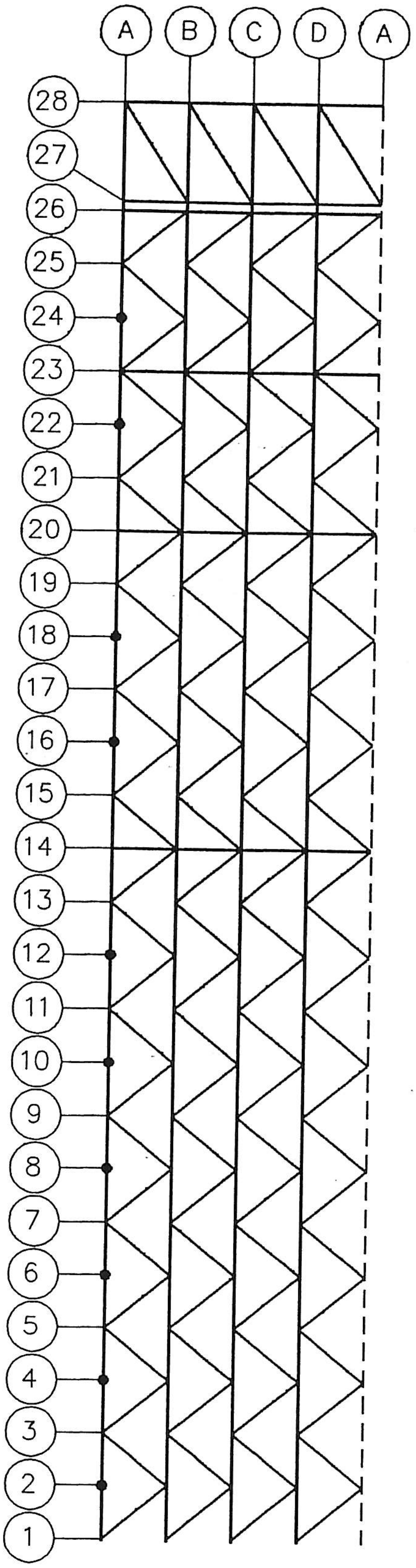
ENCLOSURE B

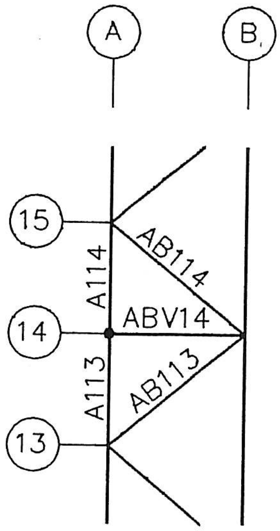
Instrumentation Diagram


ACC 1.1
ACC 1.2
ACC 1.3
ACC 2.1
ACC 2.2
ACC 2.3
CUP ANEMOMETER
WIND VANE
POWER SUPPLY
POWER SUPPLY
POWER SUPPLY
TAPE-RECORDER
KYOWA RPT-600B
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
ROCKLAND TYPE-2382
ROCKLAND TYPE-2382
ROCKLAND TYPE-2342
ROCKLAND TYPE-2582
ROCKLAND TYPE-2582
ROCKLAND TYPE-2782
PC-386
STDI
A-D CONVERTER DT2829

ENCLOSURE C

Coding Diagram for the FEM of the Mast

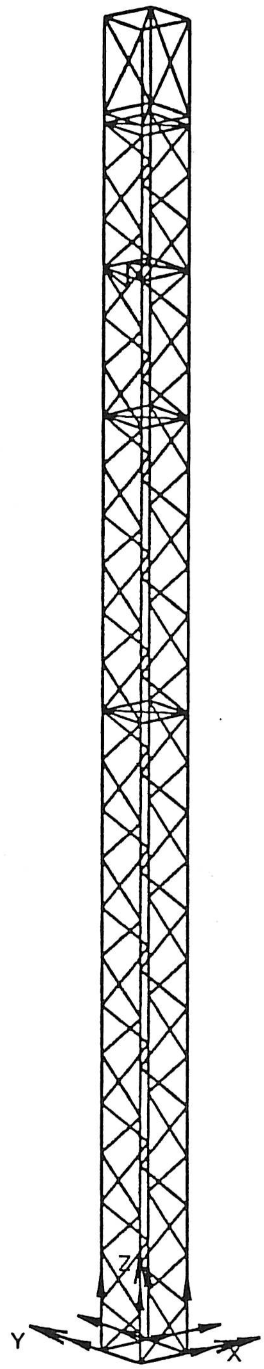




PRINCIPLE FOR
ELEMENT NAMES, 1:50

ENCLOSURE D

Mode Shapes



DIRECTION: X: 0.610 Y: 0.745 Z: -0.271
LIMITS: E 1* X(0.0000, 0.0900) Y(0.0000, 0.0900) Z(0.0000, 2.0100)

COMPANY : RAMBØLL, HANNEMANN & HØJLUND A/S

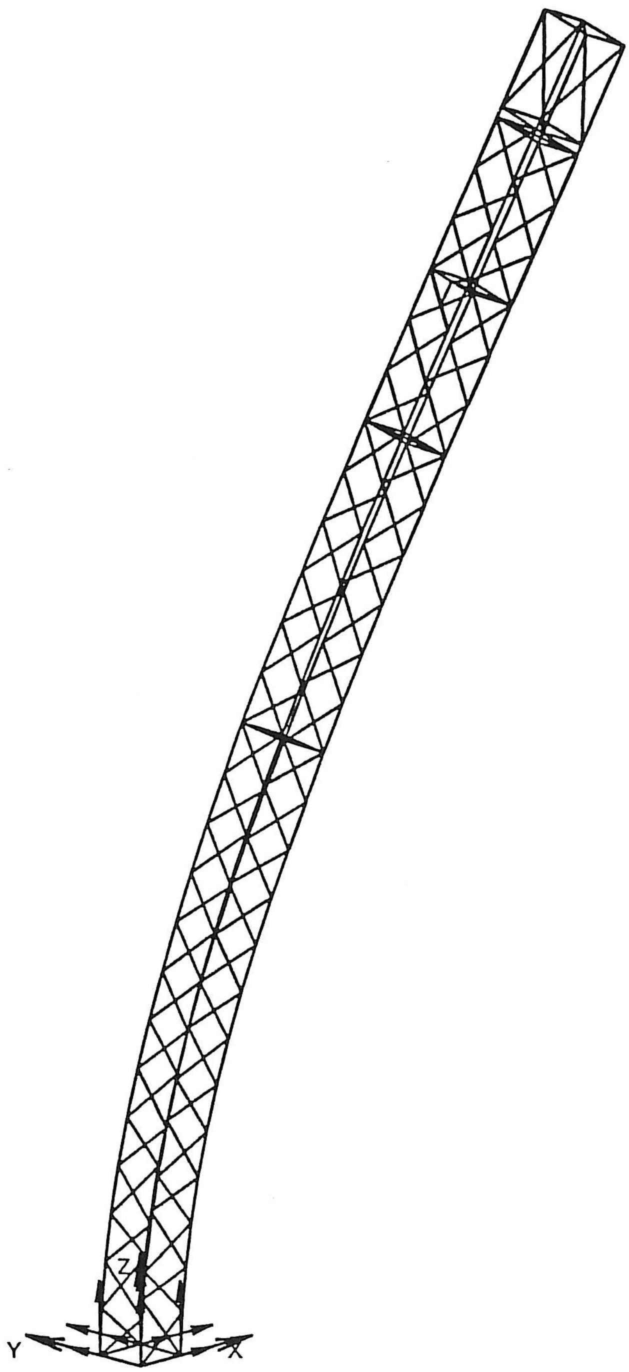
PROJECT : STÅLGITTERMAST

SUBJECT : USKADET KONSTRUKTION

DATE: 29-OCT-92 TIME: 13:36:57 JOB PHNPOE EIGENVECT. 0

RAMBØLL & HANNEMANN
RÅDGIVENDE INGENIØRER A/S

TEKNIKERBYEN 38
DK 2830 VIRUM
TELEFON 02 85 65 00



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LIMITS: E 1* X(0.0000, 1.0900) Y(-0.0187, 0.0920) Z(0.0000, 2.0403)

COMPANY : RAMBØLL, HANNEMANN & HØJLUND A/S

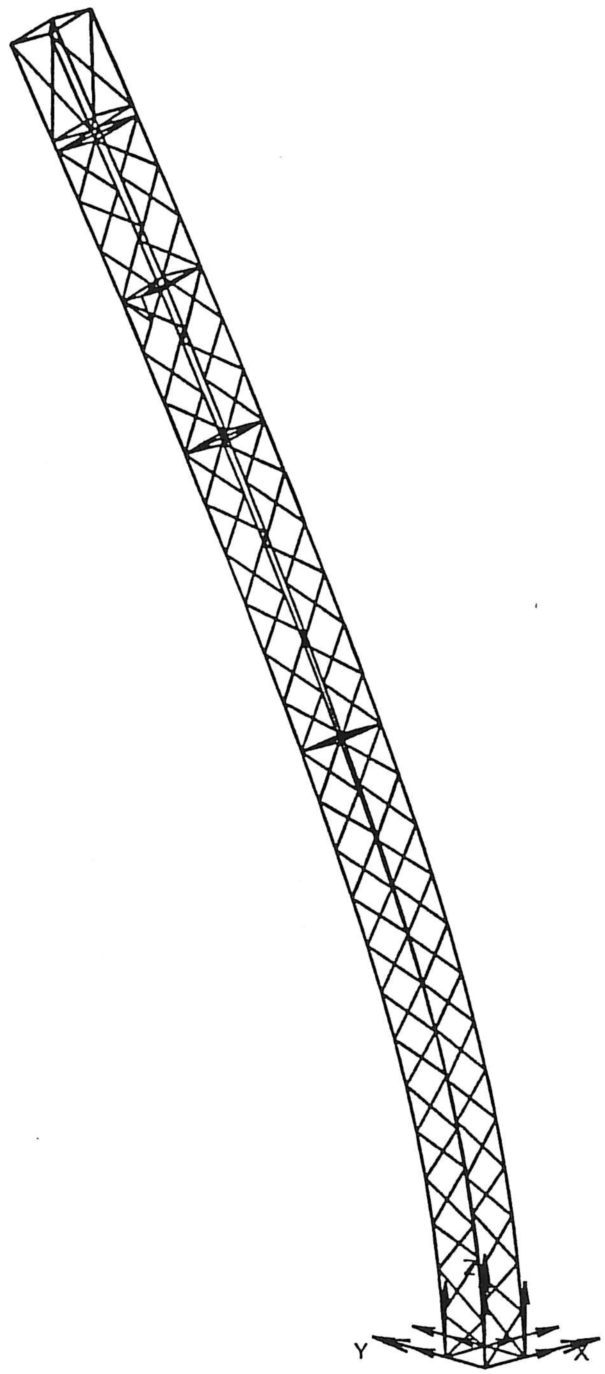
PROJECT : STÅLGITTERMAST

SUBJECT : USKADET KONSTRUKTION

DATE: 29-OCT-92 TIME: 14:45:50 JOB PHOTAC EIGENVECT. 1

RAMBØLL & HANNEMANN
RÅDGIVENDE INGENIØRER A/S

TEKNIKERBYEN 38
DK 2830 VIRUM
TELEFON 02 85 65 00



DIRECTION: X: 0.680 Y: 0.680 Z: -0.272
LIMITS: E 1* X(-0.0023, 0.1090) Y(0.0000, 1.0904) Z(0.0000, 2.0405)

COMPANY : RAMBØLL, HANNEMANN & HØJLUND A/S

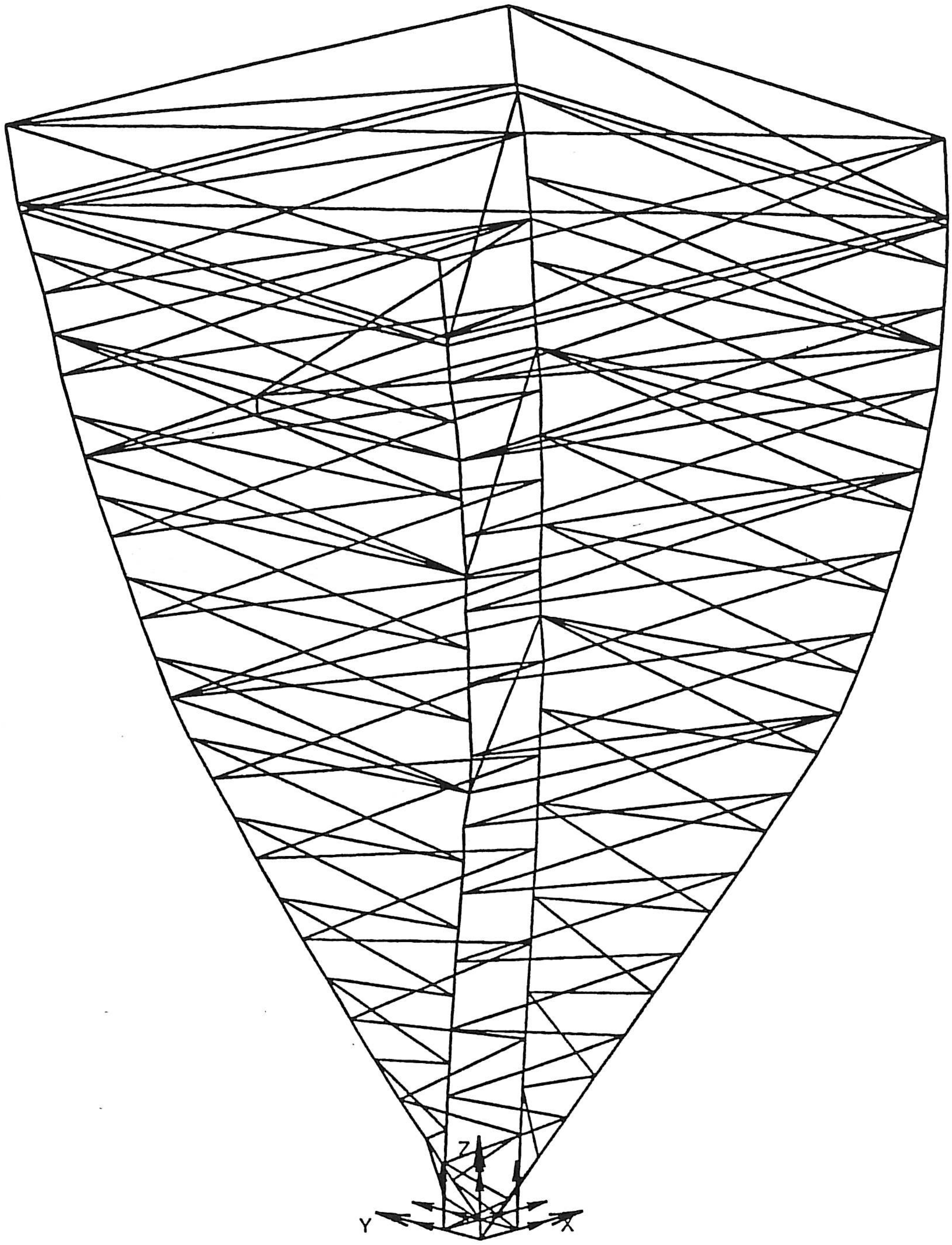
PROJECT : STÅLGITTERMAST

SUBJECT : USKADET KONSTRUKTION

DATE: 29-OCT-92 TIME: 14:45:51 JOB PHOTAC EIGENVECT. 2

RAMBØLL & HANNEMANN
RÅDGIVENDE INGENIØRER A/S

TEKNIKERBYEN 38
DK 2830 VIRUM
TELEFON 02 85 65 00



DIRECTION: X: 0.680 Y: 0.680 Z: -0.272
LIMITS: E 1* X(-0.5583, 0.6859) Y(-0.5828, 0.6638) Z(0.0000, 2.0125)

COMPANY : RAMBØLL, HANNEMANN & HØJLUND A/S

PROJECT : STÅLGITTERMAST

SUBJECT : USKADET KONSTRUKTION

DATE: 29-OCT-92 TIME: 14:45:52 JOB PHOTAC EIGENVECT. 3

RAMBØLL & HANNEMANN
RÅDGIVENDE INGENIØRER A/S

TEKNIKERBYEN 38
DK 2830 VIRUM
TELEFON 02 85 65 00

ENCLOSURE E

List of Measurements

Date	Measurement Number	Damage	Temp. ^oC
15-12-92	1	no	4
04-01-93	2	no	-5
05-01-93	3	no	-2
05-01-93	4	yes	-2
06-01-93	5	no	2
06-01-93	6	yes	2
13-01-93	7	no	3
15-01-93	8	no	2
18-01-93	9	no	4
22-01-93	10	no	3
22-01-93	11	no	3
16-03-93	12	no	8
16-03-93	13	no	6
17-03-93	14	no	8
19-03-93	15	no	9
17-05-93	16	no	15
18-05-93	17	no	20
25-05-93	18	no	16
02-06-93	19	no	17
07-06-93	20	no	16
11-06-93	21	no	22
14-06-93	22	no	16

ENCLOSURE F

Standard Deviation of Signals from Accelerometers 1.1 and 1.2, Wind-Direction and Wind-Speed (Undamaged Mast)

Measurement Number	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
1.1	0.414	0.466	1.076	6.852
1.2	0.506	0.447	1.179	6.921
1.3	0.492	0.563	1.026	7.909
1.4	0.601	0.711	1.133	7.566
1.5	0.561	0.671	0.995	8.265
1.6	0.355	0.391	0.944	6.388
1.7	0.527	0.483	1.021	6.927
1.8	0.337	0.408	1.014	6.100
1.9	0.422	0.449	1.227	7.044
1.10	0.341	0.448	1.056	7.151
2.1	0.320	0.280	3.380	7.313
2.2	0.153	0.156	3.617	6.659
2.3	0.208	0.203	4.649	7.011
2.4	0.152	0.140	4.673	6.287
2.5	0.145	0.153	4.473	5.950
2.6	0.194	0.190	4.174	6.441
2.7	0.212	0.183	4.127	6.869
2.8	0.162	0.152	4.400	6.578
2.9	0.220	0.221	4.368	6.692
2.10	0.154	0.163	4.373	6.426
3.1	0.212	0.208	2.828	6.494
3.2	0.233	0.210	2.807	6.711
3.3	0.221	0.221	2.183	6.554
3.4	0.207	0.203	2.300	6.623
3.5	0.234	0.234	2.438	7.063
3.6	0.291	0.277	2.370	7.363
3.7	0.283	0.257	2.689	7.523
3.8	0.300	0.266	2.516	7.593
3.9	0.323	0.297	2.543	7.680
3.10	0.356	0.332	2.580	7.948
5.1	0.538	0.690	1.614	7.498
5.2	0.503	0.526	1.577	7.389
5.3	0.459	0.555	1.553	6.774
5.4	0.597	0.573	1.609	8.290
5.5	0.466	0.525	1.574	7.051
5.6	0.421	0.490	1.541	6.631
5.7	0.499	0.501	1.645	7.334
5.8	0.511	0.573	1.686	6.805
5.9	0.492	0.531	1.676	6.663
5.10	0.544	0.648	1.635	8.029

Measurement Number	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
7.1	0.601	0.732	1.001	9.834
7.2	0.566	0.559	1.049	9.577
7.3	0.842	0.865	1.090	11.573
7.4	0.766	0.955	1.039	10.738
7.5	0.744	0.800	1.031	10.450
7.6	0.624	0.600	1.052	9.594
7.7	0.750	0.929	1.174	11.272
7.8	0.584	0.617	1.154	9.280
7.9	0.838	0.882	1.101	10.706
7.10	0.486	0.581	1.093	9.433
8.1	0.633	0.771	2.339	8.455
8.2	0.683	0.629	2.271	8.430
8.3	0.918	1.018	2.160	9.004
8.4	0.869	0.948	2.144	9.058
8.5	0.654	0.740	2.231	8.455
8.6	0.620	0.740	2.265	8.488
8.7	0.757	0.773	1.892	8.667
8.8	0.964	0.955	2.143	8.454
8.9	0.477	0.465	2.362	6.822
8.10	0.635	0.738	2.233	7.671
9.1	1.321	1.222	1.408	10.570
9.2	0.936	1.148	1.440	10.023
9.3	0.685	0.776	1.498	10.018
9.4	1.097	1.195	1.366	10.917
9.5	1.103	1.450	1.351	12.158
9.6	0.947	1.147	1.332	11.661
9.7	1.166	1.320	1.259	11.590
9.8	0.834	0.870	1.483	10.631
9.9	1.088	1.380	1.402	11.506
9.10	1.258	1.210	1.338	11.185
10.1	3.121	4.538	1.362	17.425
10.2	2.187	2.794	1.476	13.870
10.3	3.410	4.056	1.385	15.882
10.4	3.140	3.491	1.519	14.574
10.5	2.667	3.130	1.341	15.718
10.6	3.134	4.254	1.371	16.542
10.7	3.951	4.422	1.335	18.251
10.8	2.607	3.797	1.366	16.976
10.9	2.756	2.962	1.421	15.923
10.10	3.199	4.054	1.398	17.431

Measurement Number	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
11.1	3.412	4.338	1.367	16.731
11.2	2.116	2.184	1.383	14.307
11.3	2.848	3.760	1.378	16.087
11.4	2.659	3.231	1.434	13.215
11.5	1.730	2.161	1.457	12.783
11.6	1.649	1.723	1.445	11.616
11.7	1.857	2.332	1.414	12.501
11.8	2.666	3.357	1.385	14.729
11.9	1.475	1.715	1.475	12.534
11.10	2.454	2.376	1.381	13.859
12.1	0.047	0.534	1.415	6.840
12.2	0.015	0.451	1.370	6.725
12.3	0.015	0.568	1.495	5.285
12.4	0.013	0.391	1.465	5.890
12.5	0.027	0.803	1.460	8.570
12.6	0.011	0.396	1.430	6.505
12.7	0.015	0.499	1.460	6.220
12.8	0.015	0.528	1.510	6.745
12.9	0.023	0.862	1.415	7.380
12.10	0.015	0.461	1.570	5.455
13.1	0.015	0.546	1.444	6.484
13.2	0.020	0.631	1.436	6.352
13.3	0.022	0.764	1.405	7.360
13.4	0.018	0.567	1.425	7.140
13.5	0.016	0.494	1.522	6.143
13.6	0.022	0.764	1.387	7.925
13.7	0.019	0.649	1.391	8.392
13.8	0.021	0.872	1.414	8.117
13.9	0.019	0.794	1.376	8.140
13.10	0.019	0.603	1.409	7.696
14.1	0.568	0.680	1.369	7.347
14.2	0.454	0.515	1.368	6.963
14.3	0.416	0.531	1.423	6.688
14.4	0.567	0.615	1.483	8.053
14.5	0.474	0.502	1.417	7.256
14.6	0.574	0.668	1.353	7.550
14.7	0.362	0.418	1.438	6.952
14.8	0.742	0.906	1.344	8.575
14.9	0.558	0.718	1.456	7.629
14.10	0.755	0.857	1.339	9.111

Measurement Number	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
15.1	1.705	1.805	1.464	13.942
15.2	2.614	3.134	1.498	13.907
15.3	1.853	1.740	1.441	12.782
15.4	1.793	1.717	1.440	12.687
15.5	3.647	3.538	1.451	17.437
15.6	2.618	2.495	1.450	12.768
15.7	1.758	2.210	1.492	11.544
15.8	1.849	1.926	1.451	13.396
15.9	2.419	3.541	1.434	14.649
15.10	2.340	3.105	1.425	14.741
16.1	0.407	0.359	1.594	8.679
16.2	0.521	0.588	1.903	9.139
16.3	0.402	0.497	1.730	8.977
16.4	0.544	0.535	2.157	9.257
16.5	0.509	0.578	1.449	9.314
16.6	0.558	0.533	1.613	10.175
16.7	0.430	0.428	1.702	8.054
16.8	0.387	0.523	2.259	8.678
16.9	0.480	0.497	1.847	10.307
16.10	0.395	0.418	1.896	8.799
17.1	0.470	0.539	4.541	9.974
17.2	0.448	0.519	4.430	10.018
17.3	0.389	0.454	4.456	10.247
17.4	0.749	0.768	4.448	10.445
17.5	0.707	0.670	4.456	11.454
17.6	0.676	0.582	4.511	10.634
17.7	0.727	0.732	4.521	10.433
17.8	0.481	0.510	4.493	9.633
17.9	0.592	0.705	4.453	10.429
17.10	0.758	0.696	4.482	10.737
18.1	0.299	0.300	3.098	6.531
18.2	0.420	0.434	3.129	6.772
18.3	0.368	0.372	3.179	7.308
18.4	0.531	0.610	3.232	7.461
18.5	0.690	0.679	3.281	8.367
18.6	0.492	0.531	3.232	8.000
18.7	0.481	0.452	3.184	7.812
18.8	0.414	0.469	3.268	7.962
18.9	0.359	0.290	3.202	6.574
18.10	0.131	0.150	3.159	5.158

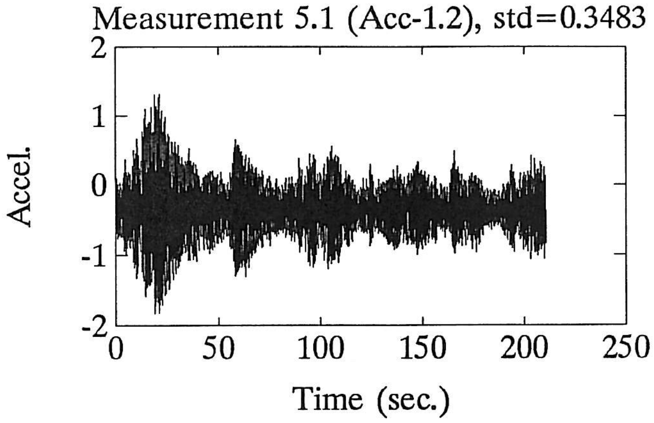
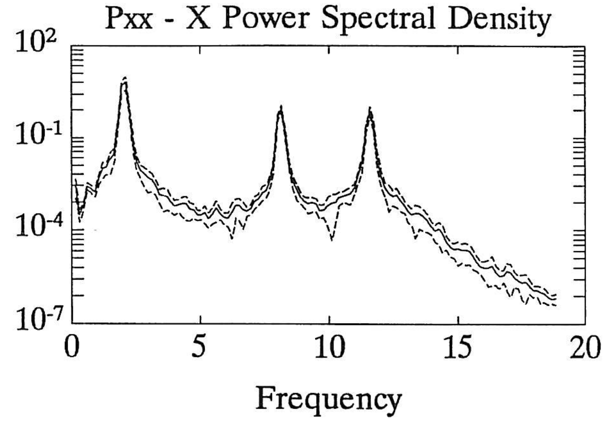
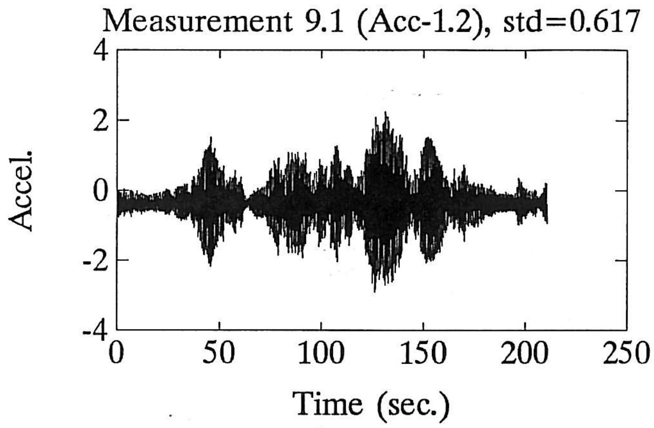
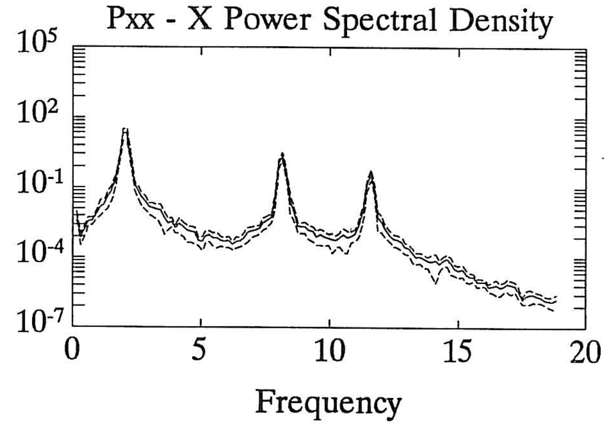
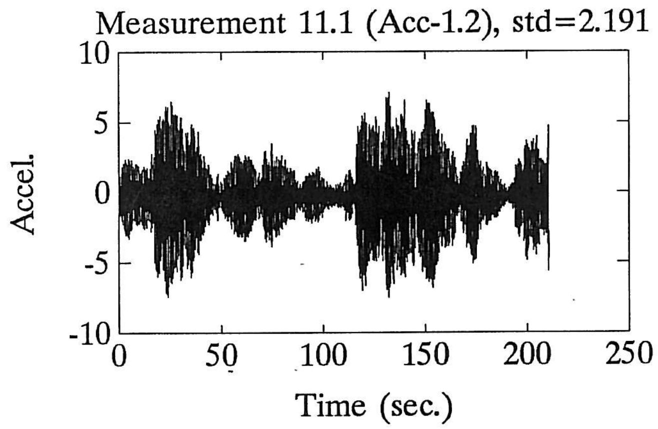
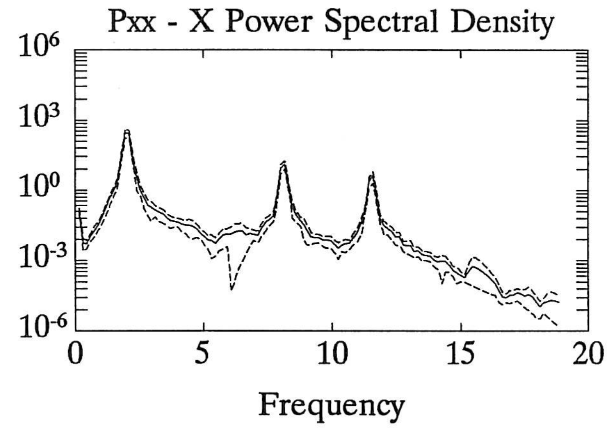
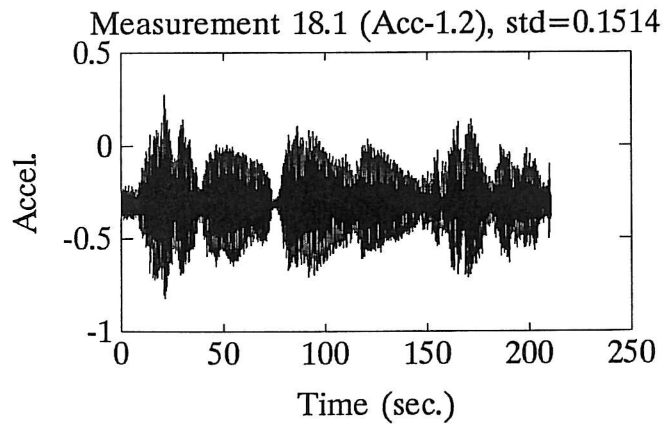
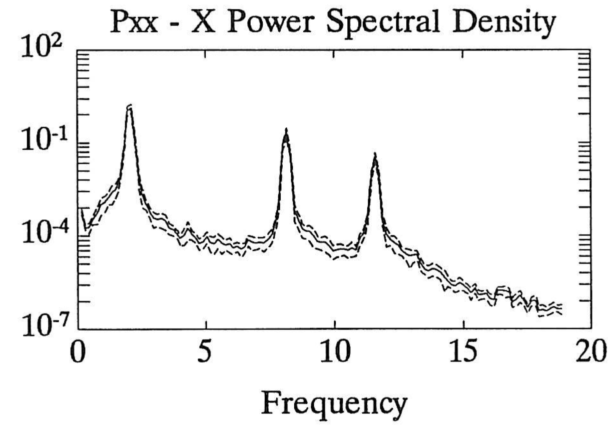
Measurement Number	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
19.1	0.215	0.199	1.713	6.845
19.2	0.427	0.439	1.449	8.852
19.3	0.357	0.399	1.443	9.078
19.4	0.428	0.454	1.614	7.380
19.5	0.463	0.481	1.360	8.973
19.6	0.218	0.244	1.531	6.410
19.7	0.206	0.225	1.552	6.792
19.8	0.356	0.366	1.480	7.751
19.9	0.396	0.458	1.395	8.842
19.10	0.232	0.270	1.697	6.469
20.1	0.170	0.145	0.107	6.663
20.2	0.112	0.117	0.109	5.068
20.3	0.150	0.167	0.110	6.921
20.4	0.153	0.140	0.110	6.784
20.5	0.195	0.185	0.111	6.238
20.6	0.121	0.156	0.111	6.049
20.7	0.150	0.162	0.112	6.209
20.8	0.148	0.148	0.111	6.677
20.9	0.138	0.134	0.112	4.939
20.10	0.182	0.196	0.112	6.685
21.1	0.100	0.097	0.126	4.862
21.2	0.066	0.067	0.124	4.342
21.3	0.046	0.055	0.124	3.492
21.4	0.091	0.109	0.123	5.066
21.5	0.085	0.094	0.122	4.883
21.6	0.064	0.067	0.119	4.058
21.7	0.112	0.106	0.119	5.078
21.8	0.072	0.074	0.118	4.028
21.9	0.115	0.117	0.118	5.021
21.10	0.091	0.074	0.118	4.564
22.1	0.312	0.357	0.126	6.915
22.2	0.428	0.391	0.125	8.215
22.3	0.592	0.597	0.124	8.224
22.4	0.685	0.898	0.123	9.677
22.5	0.734	0.730	0.121	8.464
22.6	0.603	0.509	0.120	9.416
22.7	0.555	0.765	0.120	11.054
22.8	0.523	0.706	0.119	10.446
22.9	0.629	0.714	0.117	9.972
22.10	0.581	0.573	0.118	9.935

Standard Deviation of Signals from Accelerometer 1.1 and 1.2, Wind-Direction and Wind-Speed (Damaged Mast)

Measurement Number	Damage State	Accelerometer (1.1) Std (m/s^2)	Accelerometer (1.2) Std.(m/s^2)	Wind-direction Mean (V)	Wind-speed Mean (m/s)
4.1	1	-	-	2.5265	7.9525
6.1	1	-	-	1.4233	8.0637
4.1	2	-	-	1.7135	7.4985
6.1	2	-	-	1.6024	4.9892
4.1	5	-	-	2.7452	7.5364
6.1	5	-	-	1.4756	7.2092
4.1	6	-	-	2.7195	7.6191
6.1	6	-	-	1.3835	6.1528
4.1	9	-	-	2.4895	7.4927
6.1	9	-	-	1.3603	9.0264
4.1	10	-	-	2.3310	9.2296
6.1	10	-	-	1.3446	6.2401
4.1	11	-	-	2.8363	8.3635
6.1	11	-	-	1.3536	6.1314

ENCLOSURE G

Examples of Time Series and Spectra for Different Wind Loads


Measurement 5.1 (Acc-1.2), std=0.3483
Accel.
Time (sec.)
2
1
0
-1
-2
0
50
100
150
200
250

Pxx - X Power Spectral Density
10^{2}
10^{-1}
10^{-4}
10^{-7}
Frequency
0
5
10
15
20

Measurement 9.1 (Acc-1.2), std=0.617
Accel.
Time (sec.)
4
2
0
-2
-4
0
50
100
150
200
250

Pxx - X Power Spectral Density
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
Frequency
0
5
10
15
20

Measurement 11.1 (Acc-1.2), std=2.191
Accel.
Time (sec.)
10
5
0
-5
-10
0
50
100
150
200
250

Pxx - X Power Spectral Density
10^{6}
10^{3}
10^{0}
10^{-3}
10^{-6}
Frequency
0
5
10
15
20

Measurement 18.1 (Acc-1.2), std=0.1514
Accel.
Time (sec.)
0.5
0
-0.5
-1
0
50
100
150
200
250

Pxx - X Power Spectral Density
10^{2}
10^{-1}
10^{-4}
10^{-7}
Frequency
0
5
10
15
20

ENCLOSURE H

Output from STDI

```
*************************************************************************
*                                                                       *
*                                                                       *
*             SSSSSSSSSS   TTTTTTTTTTTTTT    DDDDDDDD       II          *
*             SS                 TT          DD      DD     II          *
*             SS                 TT          DD      DD     II          *
*             SSSSSSSSSS         TT          DD      DD     II          *
*                     SS         TT          DD      DD     II          *
*                     SS         TT          DD      DD     II          *
*             SSSSSSSSSS         TT          DDDDDDDD       II          *
*                                                                       *
*                                                                       *
*************************************************************************
*                                                                       *
*      Program for Time Domain Identification of Structural Systems     *
*                    Version 0.1   August 1992                          *
*                                                                       *
*                   Developed by P.H. Kirkegaard                        *
*                       Aalborg University                              *
*                                                                       *
*************************************************************************
```

PROJECT: dam2_1

SUBJECT: Damage state 2

MEASUREMENT NUMBER: 1

USER: P.H. Kirkegaard

DATE: 27-Jul-93

TIME: 14.29.4

Program: STDI Version: 0.1 August 1992	PROJECT: dam2_1 SUBJECT: Damage state 2 MEASUREMENT NUMBER: 1 USER: P.H. Kirkegaard DATE: 27-Jul-93 TIME: 14.29.4

COMMENTS:

Temperature yesterday: -4 C
Temperature last night: -8 C

Measurement Conditions:

Damagetype	Temp
2	-2

Data-Recorder (KYOWA RPT-600B):

Speed	Tape	Start	Stop
1.2	4	0020	0145

Channel	Level	Gain	Shift	Zero	Calibration
1 (acc11)	1	5	0	0	100 % DC
2 (acc12)	1	5	0	0	100 % DC
3 (acc13)	1	5	0	0	100 % DC
4 (acc21)	1	5	0	0	100 % DC
5 (acc22)	1	5	0	0	100 % DC
6 (acc23)	1	5	0	0	100 % DC
7 (wind-dir)	6	1	0	0	100 % DC
8 (wind-speed)	4	1	0	0	100 % DC

Analog-Filters (Rockland):

Channel	Type	Setup	Cut-off freq.
1	Butterwoth 2382	LP	13.5
2	Butterwoth 2382	LP	13.5
3	Butterwoth 2342	LP	
4	Elliptic 2782	LP	
5	Bessel 2582	LP	
6	Bessel 2582	LP	

Program: STDI Version: 0.1 August 1992	PROJECT: dam2_1 SUBJECT: Damage state 2 MEASUREMENT NUMBER: 1 USER: P.H. Kirkegaard DATE: 27-Jul-93 TIME: 14.29.4

DATA ACQUISTION: (DT2829)

Number of Channels: 4

Number of Points: 8000

Sampling Ratio: 38

Date: 7-Jan-93

Time: 9.35.38.35

Channel	Type
1	acc11
2	acc12
3	wind-dir
4	wind-speed

Program: STDI Version: 0.1 August 1992	PROJECT: dam2_1 SUBJECT: Damage state 2 MEASUREMENT NUMBER: 1 USER: P.H. Kirkegaard DATE: 27-Jul-93 TIME: 14.29.4

SIGNAL PROCESSING:

Setup Number :1

Calibration:

Channel	Factor
1	1.98
2	1.98
3	6
4	32

Detrending:

Channel	Detrended
1	YES
2	YES
3	NO
4	NO

Filtring:

************ Measured Data have not been filtred ************

Chose Data Points:

************ Original number of data ************

Program: STDI Version: 0.1 August 1992	PROJECT: dam2_1 SUBJECT: Damage state 2 MEASUREMENT NUMBER: 1 USER: P.H. Kirkegaard DATE: 27-Jul-93 TIME: 14.29.4

IDENTIFICATION RESULTS:

Identification results was created on 13-1 1993 at 16:24

Data from channel: 1

Signal processing setup: 1

Identification method: ARMAX(6,5)

Number of iterations: 50

Tolerance: 1e-07

Sampling interval [s]: 0.02632

Loss function: 0.001755

Akaike's FPE: 0.00176

Akaike's AIC: 2.632e-06

Modal Parameters

Mode	Eigenfrequency Mean Hz.	Eigenfrequency Std.	Damping ratio Mean	Damping ratio Std.
-1x-	2.0017	(0.0017)	0.0017	(0.0009)
1rot	7.4477	(0.0024)	0.0010	(0.0003)
-2x-	10.8242	(0.0044)	0.0021	(0.0004)

ENCLOSURE I

Estimated Modal Parameters (Undamaged Mast)

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
1.1	1	2.0153	0.0024	0.0038	0.0012
1.1	3	8.1354	0.0031	0.0011	0.0004
1.1	4	11.4830	0.0051	0.0026	0.0004
1.2	1	2.0122	0.0019	0.0027	0.0009
1.2	3	8.1414	0.0019	0.0005	0.0002
1.2	4	11.4858	0.0050	0.0028	0.0004
1.3	1	2.0285	0.0025	0.0044	0.0012
1.3	3	8.1380	0.0021	0.0004	0.0003
1.3	4	11.4788	0.0046	0.0022	0.0004
1.4	1	2.0335	0.0022	0.0022	0.0011
1.4	3	8.1346	0.0027	0.0012	0.0003
1.4	4	11.4762	0.0043	0.0018	0.0004
1.5	1	2.0249	0.0025	0.0045	0.0012
1.5	3	8.1442	0.0021	0.0008	0.0003
1.5	4	11.4915	0.0046	0.0022	0.0004
1.6	1	2.0253	0.0021	0.0031	0.0010
1.6	3	8.1452	0.0025	0.0009	0.0003
1.6	4	11.4641	0.0039	0.0017	0.0003
1.7	1	2.0206	0.0020	0.0029	0.0010
1.7	3	8.1399	0.0023	0.0008	0.0003
1.7	4	11.4901	0.0041	0.0018	0.0004
1.8	1	2.0250	0.0022	0.0026	0.0011
1.8	3	8.1434	0.0024	0.0008	0.0003
1.8	4	11.4808	0.0035	0.0015	0.0003
1.9	1	2.0271	0.0026	0.0048	0.0013
1.9	3	8.1393	0.0017	0.0004	0.0002
1.9	4	11.4693	0.0031	0.0011	0.0003
1.10	1	2.0278	0.0030	0.0058	0.0015
1.10	3	8.1426	0.0023	0.0007	0.0003
1.10	4	11.4795	0.0051	0.0025	0.0004
1.1	2	2.0194	0.0022	0.0031	0.0011
1.1	3	8.1352	0.0032	0.0016	0.0004
1.1	5	11.5954	0.0037	0.0015	0.0003
1.2	2	2.0143	0.0021	0.0032	0.0010
1.2	3	8.1401	0.0019	0.0004	0.0002
1.2	5	11.5834	0.0031	0.0012	0.0003
1.3	2	2.0284	0.0021	0.0025	0.0010
1.3	3	8.1365	0.0020	0.0005	0.0003
1.3	5	11.5917	0.0038	0.0015	0.0003
1.4	2	2.0338	0.0017	0.0028	0.0009
1.4	3	8.1330	0.0028	0.0012	0.0003
1.4	5	11.5876	0.0036	0.0015	0.0003
1.5	2	2.0244	0.0019	0.0025	0.0009
1.5	3	8.1418	0.0021	0.0007	0.0003
1.5	5	11.5921	0.0033	0.0012	0.0003
1.6	2	2.0257	0.0018	0.0022	0.0009
1.6	3	8.1427	0.0025	0.0009	0.0003
1.6	5	11.5820	0.0036	0.0017	0.0003
1.7	2	2.0227	0.0021	0.0028	0.0010
1.7	3	8.1391	0.0023	0.0007	0.0003
1.7	5	11.5852	0.0025	0.0009	0.0002
1.8	2	2.0258	0.0018	0.0023	0.0009
1.8	3	8.1422	0.0025	0.0010	0.0003
1.8	5	11.5894	0.0026	0.0010	0.0002
1.9	2	2.0269	0.0021	0.0028	0.0010
1.9	3	8.1379	0.0016	0.0005	0.0002
1.9	5	11.5890	0.0039	0.0018	0.0003
1.10	2	2.0247	0.0019	0.0027	0.0009
1.10	3	8.1410	0.0023	0.0009	0.0003
1.10	5	11.5889	0.0038	0.0015	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
2.1	1	2.0272	0.0021	0.0025	0.0010
2.1	3	8.1565	0.0015	0.0003	0.0002
2.1	4	11.5126	0.0023	0.0009	0.0002
2.2	1	2.0364	0.0032	0.0062	0.0016
2.2	3	8.1697	0.0033	0.0017	0.0004
2.2	4	11.5166	0.0037	0.0018	0.0003
2.3	1	2.0275	0.0020	0.0025	0.0010
2.3	3	8.1646	0.0025	0.0010	0.0003
2.3	4	11.5202	0.0037	0.0020	0.0003
2.4	1	2.0185	0.0025	0.0041	0.0012
2.4	3	8.1631	0.0027	0.0013	0.0003
2.4	4	11.5117	0.0038	0.0015	0.0003
2.5	1	2.0232	0.0036	0.0078	0.0018
2.5	4	8.1599	0.0017	0.0004	0.0002
2.5	5	11.5249	0.0033	0.0012	0.0003
2.6	1	2.0280	0.0028	0.0051	0.0014
2.6	3	8.1595	0.0017	0.0005	0.0002
2.6	4	11.5146	0.0039	0.0015	0.0003
2.7	1	2.0245	0.0031	0.0055	0.0015
2.7	3	8.1676	0.0026	0.0010	0.0003
2.7	4	11.5188	0.0024	0.0010	0.0002
2.8	1	2.0275	0.0025	0.0038	0.0012
2.8	3	8.1501	0.0023	0.0008	0.0003
2.8	4	11.5177	0.0031	0.0010	0.0003
2.9	1	2.0251	0.0021	0.0023	0.0010
2.9	3	8.1614	0.0026	0.0009	0.0003
2.9	4	11.5214	0.0041	0.0018	0.0004
2.10	1	2.0326	0.0027	0.0046	0.0013
2.10	3	8.1683	0.0039	0.0025	0.0005
2.10	4	11.5242	0.0039	0.0018	0.0003
2.1	2	2.0276	0.0023	0.0026	0.0011
2.1	3	8.1557	0.0014	0.0003	0.0002
2.1	5	11.5790	0.0039	0.0017	0.0003
2.2	2	2.0380	0.0034	0.0069	0.0016
2.2	3	8.1505	0.0026	0.0012	0.0003
2.2	5	11.6292	0.0047	0.0027	0.0004
2.3	2	2.0338	0.0027	0.0042	0.0013
2.3	3	8.1624	0.0023	0.0007	0.0003
2.3	5	11.6456	0.0033	0.0014	0.0003
2.4	2	2.0224	0.0031	0.0063	0.0016
2.4	3	8.1501	0.0023	0.0010	0.0003
2.4	5	11.6254	0.0053	0.0031	0.0005
2.5	2	2.0306	0.0035	0.0083	0.0017
2.5	3	8.1583	0.0016	0.0005	0.0002
2.5	5	11.6451	0.0039	0.0016	0.0003
2.6	2	2.0251	0.0034	0.0070	0.0017
2.6	3	8.1580	0.0015	0.0004	0.0002
2.6	5	11.6151	0.0053	0.0028	0.0005
2.7	2	2.0292	0.0039	0.0104	0.0019
2.7	3	8.1636	0.0023	0.0008	0.0003
2.7	5	11.6076	0.0032	0.0022	0.0003
2.8	2	2.0271	0.0037	0.0069	0.0018
2.8	3	8.1571	0.0019	0.0007	0.0002
2.8	5	11.6330	0.0040	0.0017	0.0003
2.9	2	2.0328	0.0026	0.0045	0.0013
2.9	3	8.1585	0.0022	0.0006	0.0003
2.9	5	11.6361	0.0039	0.0014	0.0003
2.10	2	2.0354	0.0033	0.0066	0.0016
2.10	3	8.1500	0.0033	0.0015	0.0004
2.10	5	11.6333	0.0048	0.0025	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
3.1	1	2.0257	0.0030	0.0050	0.0015
3.1	3	8.1654	0.0020	0.0006	0.0002
3.1	4	11.5257	0.0022	0.0005	0.0002
3.2	1	2.0286	0.0029	0.0055	0.0014
3.2	3	8.1651	0.0018	0.0006	0.0002
3.2	4	11.5324	0.0030	0.0011	0.0003
3.3	1	2.0312	0.0035	0.0068	0.0017
3.3	3	8.1675	0.0020	0.0008	0.0002
3.3	4	11.5263	0.0039	0.0016	0.0003
3.4	1	2.0294	0.0044	0.0114	0.0022
3.4	3	8.1635	0.0018	0.0005	0.0002
3.4	4	11.5227	0.0041	0.0020	0.0004
3.5	1	2.0295	0.0024	0.0034	0.0012
3.5	3	8.1688	0.0031	0.0012	0.0004
3.5	4	11.5250	0.0037	0.0016	0.0003
3.6	1	2.0309	0.0027	0.0048	0.0013
3.6	3	8.1640	0.0019	0.0008	0.0002
3.6	4	11.5277	0.0036	0.0016	0.0003
3.7	1	2.0318	0.0027	0.0042	0.0014
3.7	3	8.1712	0.0024	0.0008	0.0003
3.7	4	11.5254	0.0037	0.0014	0.0003
3.8	1	2.0306	0.0030	0.0054	0.0015
3.8	3	8.1652	0.0019	0.0006	0.0002
3.8	4	11.5167	0.0022	0.0006	0.0002
3.9	1	2.0371	0.0030	0.0049	0.0015
3.9	3	8.1631	0.0020	0.0006	0.0002
3.9	4	11.5214	0.0027	0.0010	0.0002
3.10	1	2.0318	0.0026	0.0039	0.0013
3.10	3	8.1659	0.0017	0.0005	0.0002
3.10	4	11.5191	0.0039	0.0017	0.0003
3.1	2	2.0329	0.0034	0.0080	0.0017
3.1	3	8.1625	0.0017	0.0004	0.0002
3.1	5	11.6232	0.0037	0.0015	0.0003
3.2	2	2.0336	0.0038	0.0097	0.0019
3.2	3	8.1626	0.0016	0.0004	0.0002
3.2	5	11.6537	0.0032	0.0011	0.0003
3.3	2	2.0346	0.0034	0.0065	0.0017
3.3	3	8.1660	0.0017	0.0006	0.0002
3.3	5	11.6437	0.0037	0.0014	0.0003
3.4	2	2.0360	0.0048	0.0139	0.0024
3.4	3	8.1615	0.0015	0.0005	0.0002
3.4	5	11.6345	0.0050	0.0023	0.0004
3.5	2	2.0311	0.0026	0.0046	0.0013
3.5	3	8.1641	0.0027	0.0007	0.0003
3.5	5	11.6219	0.0044	0.0023	0.0004
3.6	2	2.0376	0.0032	0.0062	0.0016
3.6	3	8.1617	0.0016	0.0006	0.0002
3.6	5	11.6312	0.0042	0.0020	0.0004
3.7	2	2.0358	0.0031	0.0057	0.0015
3.7	3	8.1686	0.0023	0.0009	0.0003
3.7	5	11.6264	0.0037	0.0014	0.0003
3.8	2	2.0368	0.0031	0.0063	0.0015
3.8	3	8.1636	0.0018	0.0005	0.0002
3.8	5	11.5732	0.0037	0.0017	0.0003
3.9	2	2.0433	0.0029	0.0053	0.0014
3.9	3	8.1617	0.0018	0.0006	0.0002
3.9	5	11.6106	0.0040	0.0020	0.0003
3.10	2	2.0375	0.0030	0.0044	0.0015
3.10	3	8.1642	0.0016	0.0004	0.0002
3.10	5	11.6246	0.0039	0.0019	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
5.1	1	2.0250	0.0026	0.0044	0.0013
5.1	3	8.1349	0.0024	0.0009	0.0003
5.1	4	11.4836	0.0038	0.0016	0.0003
5.2	1	2.0260	0.0026	0.0041	0.0013
5.2	3	8.1377	0.0023	0.0007	0.0003
5.2	4	11.4920	0.0057	0.0034	0.0005
5.3	1	2.0238	0.0026	0.0047	0.0013
5.3	3	8.1357	0.0028	0.0012	0.0003
5.3	4	11.5040	0.0050	0.0024	0.0004
5.4	1	2.0142	0.0023	0.0031	0.0012
5.4	3	8.1381	0.0018	0.0006	0.0002
5.4	4	11.4870	0.0046	0.0023	0.0004
5.5	1	2.0190	0.0026	0.0047	0.0013
5.5	3	8.1339	0.0019	0.0006	0.0002
5.5	4	11.4950	0.0047	0.0024	0.0004
5.6	1	2.0283	0.0027	0.0044	0.0013
5.6	3	8.1356	0.0027	0.0010	0.0003
5.6	4	11.5188	0.0037	0.0016	0.0003
5.7	1	2.0199	0.0023	0.0032	0.0011
5.7	3	8.1368	0.0020	0.0004	0.0002
5.7	4	11.5036	0.0047	0.0024	0.0004
5.8	1	2.0259	0.0024	0.0037	0.0012
5.8	3	8.1337	0.0023	0.0008	0.0003
5.8	4	11.4991	0.0041	0.0021	0.0004
5.9	1	2.0211	0.0024	0.0028	0.0012
5.9	3	8.1383	0.0019	0.0005	0.0002
5.9	4	11.4852	0.0039	0.0018	0.0003
5.10	1	2.0228	0.0023	0.0034	0.0011
5.10	3	8.1404	0.0027	0.0012	0.0003
5.10	4	11.5158	0.0044	0.0022	0.0004
5.1	2	2.0282	0.0020	0.0032	0.0010
5.1	3	8.1320	0.0023	0.0008	0.0003
5.1	5	11.5983	0.0034	0.0013	0.0003
5.2	2	2.0301	0.0024	0.0044	0.0012
5.2	3	8.1340	0.0023	0.0007	0.0003
5.2	5	11.6159	0.0040	0.0019	0.0003
5.3	2	2.0241	0.0020	0.0028	0.0010
5.3	3	8.1335	0.0028	0.0012	0.0003
5.3	5	11.6075	0.0033	0.0015	0.0003
5.4	2	2.0133	0.0026	0.0050	0.0013
5.4	3	8.1358	0.0018	0.0005	0.0002
5.4	5	11.6018	0.0040	0.0017	0.0003
5.5	2	2.0181	0.0024	0.0038	0.0012
5.5	3	8.1318	0.0020	0.0007	0.0002
5.5	5	11.6073	0.0027	0.0010	0.0002
5.6	2	2.0263	0.0024	0.0036	0.0012
5.6	3	8.1323	0.0028	0.0011	0.0003
5.6	5	11.5988	0.0024	0.0009	0.0002
5.7	2	2.0311	0.0024	0.0047	0.0012
5.7	3	8.1342	0.0020	0.0007	0.0002
5.7	5	11.6012	0.0033	0.0014	0.0003
5.8	2	2.0277	0.0020	0.0024	0.0010
5.8	3	8.1319	0.0025	0.0008	0.0003
5.8	5	11.5980	0.0032	0.0011	0.0003
5.9	2	2.0266	0.0021	0.0034	0.0010
5.9	3	8.1360	0.0020	0.0006	0.0003
5.9	5	11.5924	0.0035	0.0014	0.0003
5.10	2	2.0211	0.0021	0.0021	0.0010
5.10	3	8.1372	0.0029	0.0015	0.0004
5.10	5	11.6046	0.0030	0.0011	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
7.1	1	2.0265	0.0024	0.0040	0.0012
7.1	3	8.1415	0.0024	0.0008	0.0003
7.1	4	11.4823	0.0048	0.0026	0.0004
7.2	1	2.0238	0.0024	0.0028	0.0012
7.2	3	8.1348	0.0023	0.0009	0.0003
7.2	4	11.4861	0.0050	0.0028	0.0004
7.3	1	2.0205	0.0025	0.0036	0.0013
7.3	3	8.1391	0.0018	0.0005	0.0002
7.3	4	11.4627	0.0044	0.0020	0.0004
7.4	1	2.0268	0.0027	0.0045	0.0013
7.4	3	8.1391	0.0018	0.0006	0.0002
7.4	4	11.4853	0.0052	0.0030	0.0005
7.5	1	2.0247	0.0023	0.0042	0.0011
7.5	3	8.1401	0.0025	0.0008	0.0003
7.5	4	11.4649	0.0043	0.0018	0.0004
7.6	1	2.0249	0.0024	0.0039	0.0012
7.6	3	8.1360	0.0028	0.0013	0.0003
7.6	4	11.4724	0.0040	0.0017	0.0003
7.7	1	2.0180	0.0028	0.0049	0.0014
7.7	3	8.1413	0.0027	0.0011	0.0003
7.7	4	11.4782	0.0045	0.0021	0.0004
7.8	1	2.0257	0.0023	0.0028	0.0012
7.8	3	8.1384	0.0029	0.0012	0.0004
7.8	4	11.5002	0.0058	0.0033	0.0005
7.9	1	2.0270	0.0020	0.0026	0.0010
7.9	3	8.1330	0.0027	0.0011	0.0003
7.9	4	11.4981	0.0052	0.0028	0.0005
7.10	1	2.0254	0.0029	0.0056	0.0015
7.10	3	8.1370	0.0026	0.0011	0.0003
7.10	4	11.4821	0.0051	0.0027	0.0004
7.1	2	2.0295	0.0019	0.0019	0.0009
7.1	3	8.1403	0.0024	0.0008	0.0003
7.1	5	11.5789	0.0034	0.0013	0.0003
7.2	2	2.0299	0.0025	0.0047	0.0012
7.2	3	8.1341	0.0024	0.0008	0.0003
7.2	5	11.5898	0.0032	0.0012	0.0003
7.3	2	2.0240	0.0023	0.0026	0.0012
7.3	3	8.1391	0.0018	0.0006	0.0002
7.3	5	11.5821	0.0047	0.0023	0.0004
7.4	2	2.0254	0.0017	0.0024	0.0009
7.4	3	8.1387	0.0017	0.0005	0.0002
7.4	5	11.5822	0.0040	0.0017	0.0003
7.5	2	2.0254	0.0019	0.0021	0.0009
7.5	3	8.1381	0.0024	0.0008	0.0003
7.5	5	11.5839	0.0041	0.0018	0.0003
7.6	2	2.0249	0.0023	0.0034	0.0011
7.6	3	8.1341	0.0027	0.0013	0.0003
7.6	5	11.5741	0.0036	0.0014	0.0003
7.7	2	2.0209	0.0022	0.0033	0.0011
7.7	3	8.1399	0.0025	0.0011	0.0003
7.7	5	11.5864	0.0035	0.0014	0.0003
7.8	2	2.0252	0.0022	0.0028	0.0011
7.8	3	8.1380	0.0029	0.0011	0.0004
7.8	5	11.5863	0.0039	0.0018	0.0003
7.9	2	2.0269	0.0018	0.0027	0.0009
7.9	3	8.1328	0.0026	0.0009	0.0003
7.9	5	11.5811	0.0034	0.0014	0.0003
7.10	2	2.0304	0.0023	0.0037	0.0012
7.10	3	8.1361	0.0025	0.0010	0.0003
7.10	5	11.5834	0.0033	0.0016	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
8.1	1	2.0227	0.0024	0.0030	0.0012
8.1	3	8.1372	0.0025	0.0010	0.0003
8.1	4	11.4601	0.0046	0.0022	0.0004
8.2	1	2.0195	0.0025	0.0034	0.0012
8.2	3	8.1354	0.0027	0.0009	0.0003
8.2	4	11.4556	0.0038	0.0017	0.0003
8.3	1	2.0258	0.0023	0.0032	0.0011
8.3	3	8.1470	0.0021	0.0005	0.0003
8.3	4	11.4557	0.0041	0.0017	0.0004
8.4	1	2.0234	0.0021	0.0024	0.0011
8.4	3	8.1398	0.0023	0.0008	0.0003
8.4	4	11.4577	0.0050	0.0026	0.0004
8.5	1	2.0208	0.0023	0.0031	0.0011
8.5	3	8.1354	0.0024	0.0008	0.0003
8.5	4	11.4560	0.0036	0.0016	0.0003
8.6	1	2.0202	0.0029	0.0048	0.0015
8.6	3	8.1457	0.0027	0.0010	0.0003
8.6	4	11.4575	0.0040	0.0017	0.0003
8.7	1	2.0287	0.0024	0.0045	0.0012
8.7	3	8.1383	0.0015	0.0004	0.0002
8.7	4	11.4615	0.0047	0.0022	0.0004
8.8	1	2.0128	0.0019	0.0030	0.0009
8.8	3	8.1379	0.0017	0.0004	0.0002
8.8	4	11.4585	0.0033	0.0013	0.0003
8.9	1	2.0230	0.0020	0.0025	0.0010
8.9	3	8.1424	0.0020	0.0008	0.0003
8.9	4	11.4626	0.0029	0.0010	0.0003
8.10	1	2.0202	0.0021	0.0033	0.0010
8.10	3	8.1433	0.0019	0.0007	0.0002
8.10	4	11.4578	0.0044	0.0020	0.0004
8.1	2	2.0237	0.0023	0.0033	0.0011
8.1	3	8.1388	0.0025	0.0010	0.0003
8.1	5	11.5655	0.0044	0.0021	0.0004
8.2	2	2.0220	0.0033	0.0069	0.0016
8.2	3	8.1378	0.0028	0.0012	0.0003
8.2	5	11.5497	0.0048	0.0028	0.0004
8.3	2	2.0260	0.0020	0.0034	0.0010
8.3	3	8.1472	0.0022	0.0007	0.0003
8.3	5	11.5660	0.0038	0.0014	0.0003
8.4	2	2.0274	0.0022	0.0038	0.0011
8.4	3	8.1396	0.0023	0.0010	0.0003
8.4	5	11.5748	0.0043	0.0021	0.0004
8.5	2	2.0251	0.0022	0.0034	0.0011
8.5	3	8.1371	0.0024	0.0007	0.0003
8.5	5	11.5577	0.0049	0.0027	0.0004
8.6	2	2.0213	0.0027	0.0045	0.0013
8.6	3	8.1470	0.0028	0.0013	0.0003
8.6	5	11.5627	0.0037	0.0014	0.0003
8.7	2	2.0269	0.0024	0.0047	0.0012
8.7	3	8.1398	0.0016	0.0004	0.0002
8.7	5	11.5732	0.0039	0.0017	0.0003
8.8	2	2.0190	0.0022	0.0030	0.0011
8.8	3	8.1366	0.0018	0.0006	0.0002
8.8	5	11.5579	0.0043	0.0018	0.0004
8.9	2	2.0257	0.0026	0.0031	0.0013
8.9	3	8.1413	0.0020	0.0006	0.0002
8.9	5	11.5601	0.0036	0.0015	0.0003
8.10	2	2.0261	0.0021	0.0034	0.0010
8.10	3	8.1422	0.0019	0.0007	0.0002
8.10	5	11.5665	0.0052	0.0028	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
9.1	1	2.0092	0.0024	0.0039	0.0012
9.1	3	8.1406	0.0025	0.0009	0.0003
9.1	4	11.4637	0.0044	0.0016	0.0004
9.2	1	2.0143	0.0023	0.0035	0.0012
9.2	3	8.1384	0.0018	0.0008	0.0002
9.2	4	11.4553	0.0055	0.0028	0.0005
9.3	1	2.0183	0.0028	0.0042	0.0014
9.3	3	8.1359	0.0026	0.0012	0.0003
9.3	4	11.4626	0.0042	0.0019	0.0004
9.4	1	2.0035	0.0019	0.0033	0.0010
9.4	3	8.1455	0.0034	0.0014	0.0004
9.4	4	11.4744	0.0046	0.0020	0.0004
9.5	1	2.0168	0.0025	0.0048	0.0013
9.5	3	8.1364	0.0024	0.0008	0.0003
9.5	4	11.4581	0.0036	0.0012	0.0003
9.6	1	2.0205	0.0032	0.0066	0.0016
9.6	3	8.1382	0.0025	0.0014	0.0003
9.6	4	11.4572	0.0034	0.0014	0.0003
9.7	1	2.0075	0.0023	0.0040	0.0011
9.7	3	8.1392	0.0021	0.0007	0.0003
9.7	4	11.4797	0.0053	0.0031	0.0005
9.8	1	2.0168	0.0026	0.0035	0.0013
9.8	3	8.1393	0.0022	0.0009	0.0003
9.8	4	11.4583	0.0039	0.0016	0.0003
9.9	1	2.0125	0.0027	0.0053	0.0014
9.9	3	8.1325	0.0029	0.0016	0.0004
9.9	4	11.4806	0.0051	0.0028	0.0004
9.10	1	2.0106	0.0021	0.0029	0.0011
9.10	3	8.1358	0.0027	0.0012	0.0003
9.10	4	11.4460	0.0040	0.0013	0.0003
9.1	2	2.0132	0.0023	0.0025	0.0012
9.1	3	8.1396	0.0023	0.0008	0.0003
9.1	5	11.5696	0.0039	0.0015	0.0003
9.2	2	2.0140	0.0017	0.0023	0.0009
9.2	3	8.1377	0.0017	0.0006	0.0002
9.2	5	11.5860	0.0053	0.0027	0.0005
9.3	2	2.0185	0.0024	0.0056	0.0012
9.3	3	8.1343	0.0025	0.0011	0.0003
9.3	5	11.5755	0.0035	0.0013	0.0003
9.4	2	2.0107	0.0016	0.0023	0.0008
9.4	3	8.1415	0.0031	0.0014	0.0004
9.4	5	11.5718	0.0037	0.0016	0.0003
9.5	2	2.0090	0.0019	0.0022	0.0009
9.5	3	8.1340	0.0023	0.0008	0.0003
9.5	5	11.5673	0.0037	0.0015	0.0003
9.6	2	2.0153	0.0022	0.0029	0.0011
9.6	3	8.1361	0.0025	0.0012	0.0003
9.6	5	11.5766	0.0041	0.0018	0.0004
9.7	2	2.0119	0.0019	0.0022	0.0010
9.7	3	8.1370	0.0020	0.0007	0.0003
9.7	5	11.5729	0.0042	0.0018	0.0004
9.8	2	2.0180	0.0022	0.0032	0.0011
9.8	3	8.1388	0.0021	0.0010	0.0003
9.8	5	11.5787	0.0040	0.0019	0.0003
9.9	2	2.0178	0.0020	0.0037	0.0010
9.9	3	8.1304	0.0027	0.0016	0.0003
9.9	5	11.5690	0.0041	0.0019	0.0004
9.10	2	2.0099	0.0022	0.0015	0.0011
9.10	3	8.1318	0.0024	0.0011	0.0003
9.10	5	11.5715	0.0043	0.0018	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
10.1	1	2.0027	0.0028	0.0053	0.0014
10.1	3	8.1301	0.0032	0.0014	0.0004
10.1	4	11.4533	0.0073	0.0049	0.0006
10.2	1	2.0084	0.0023	0.0029	0.0011
10.2	3	8.1293	0.0026	0.0011	0.0003
10.2	4	11.4528	0.0051	0.0028	0.0004
10.3	1	1.9953	0.0020	0.0028	0.0010
10.3	3	8.1276	0.0032	0.0014	0.0004
10.3	4	11.4458	0.0067	0.0042	0.0006
10.4	1	2.0016	0.0021	0.0032	0.0011
10.4	3	8.1293	0.0035	0.0019	0.0004
10.4	4	11.4487	0.0056	0.0032	0.0005
10.5	1	2.0013	0.0028	0.0050	0.0014
10.5	3	8.1286	0.0027	0.0012	0.0003
10.5	4	11.4484	0.0056	0.0031	0.0005
10.6	1	2.0038	0.0026	0.0045	0.0013
10.6	3	8.1244	0.0027	0.0012	0.0003
10.6	4	11.4407	0.0054	0.0033	0.0005
10.7	1	2.0035	0.0022	0.0036	0.0011
10.7	3	8.1384	0.0041	0.0023	0.0005
10.7	4	11.4351	0.0054	0.0033	0.0005
10.8	1	2.0044	0.0028	0.0049	0.0014
10.8	3	8.1334	0.0032	0.0015	0.0004
10.8	4	11.4504	0.0055	0.0036	0.0005
10.9	1	2.0024	0.0022	0.0031	0.0011
10.9	3	8.1286	0.0028	0.0013	0.0003
10.9	4	11.4509	0.0055	0.0029	0.0005
10.10	1	2.0033	0.0026	0.0042	0.0013
10.10	3	8.1286	0.0027	0.0010	0.0003
10.10	4	11.4576	0.0066	0.0042	0.0006
10.1	2	2.0052	0.0018	0.0024	0.0009
10.1	3	8.1285	0.0035	0.0018	0.0004
10.1	5	11.5616	0.0066	0.0037	0.0006
10.2	2	2.0121	0.0020	0.0035	0.0010
10.2	3	8.1293	0.0025	0.0013	0.0003
10.2	5	11.5516	0.0037	0.0014	0.0003
10.3	2	1.9972	0.0017	0.0027	0.0009
10.3	3	8.1275	0.0031	0.0013	0.0004
10.3	5	11.5550	0.0057	0.0033	0.0005
10.4	2	2.0078	0.0020	0.0047	0.0010
10.4	3	8.1314	0.0037	0.0022	0.0005
10.4	5	11.5597	0.0053	0.0026	0.0005
10.5	2	2.0026	0.0023	0.0027	0.0011
10.5	3	8.1278	0.0025	0.0012	0.0003
10.5	5	11.5589	0.0057	0.0031	0.0005
10.6	2	2.0098	0.0023	0.0039	0.0012
10.6	3	8.1247	0.0027	0.0012	0.0003
10.6	5	11.5538	0.0053	0.0031	0.0005
10.7	2	2.0090	0.0020	0.0038	0.0010
10.7	3	8.1335	0.0037	0.0020	0.0004
10.7	5	11.5511	0.0063	0.0039	0.0005
10.8	2	2.0026	0.0021	0.0031	0.0010
10.8	3	8.1312	0.0030	0.0013	0.0004
10.8	5	11.5511	0.0042	0.0020	0.0004
10.9	2	2.0080	0.0023	0.0031	0.0012
10.9	3	8.1261	0.0026	0.0013	0.0003
10.9	5	11.5602	0.0048	0.0023	0.0004
10.10	2	2.0065	0.0023	0.0039	0.0011
10.10	3	8.1259	0.0025	0.0008	0.0003
10.10	5	11.5599	0.0054	0.0030	0.0005

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
11.1	1	1.9975	0.0024	0.0030	0.0012
11.1	3	8.1324	0.0032	0.0012	0.0004
11.1	4	11.4534	0.0053	0.0028	0.0005
11.2	1	2.0026	0.0024	0.0042	0.0012
11.2	3	8.1392	0.0032	0.0016	0.0004
11.2	4	11.4515	0.0050	0.0026	0.0004
11.3	1	2.0027	0.0027	0.0058	0.0013
11.3	3	8.1294	0.0028	0.0013	0.0003
11.3	4	11.4531	0.0054	0.0032	0.0005
11.4	1	2.0022	0.0020	0.0028	0.0010
11.4	3	8.1323	0.0030	0.0017	0.0004
11.4	4	11.4368	0.0054	0.0029	0.0005
11.5	1	2.0108	0.0025	0.0037	0.0012
11.5	3	8.1356	0.0027	0.0012	0.0003
11.5	4	11.4451	0.0042	0.0019	0.0004
11.6	1	2.0095	0.0021	0.0021	0.0010
11.6	3	8.1323	0.0024	0.0013	0.0003
11.6	4	11.4408	0.0042	0.0019	0.0004
11.7	1	2.0086	0.0025	0.0053	0.0013
11.7	3	8.1308	0.0028	0.0013	0.0003
11.7	4	11.4563	0.0057	0.0033	0.0005
11.8	1	1.9974	0.0020	0.0026	0.0010
11.8	3	8.1305	0.0025	0.0009	0.0003
11.8	4	11.4569	0.0060	0.0041	0.0005
11.9	1	2.0015	0.0027	0.0049	0.0014
11.9	3	8.1365	0.0023	0.0007	0.0003
11.9	4	11.4445	0.0041	0.0019	0.0004
11.10	1	2.0029	0.0021	0.0029	0.0010
11.10	3	8.1297	0.0022	0.0007	0.0003
11.10	4	11.4450	0.0044	0.0020	0.0004
11.1	2	2.0068	0.0023	0.0037	0.0012
11.1	3	8.1331	0.0031	0.0012	0.0004
11.1	5	11.5543	0.0040	0.0014	0.0003
11.2	2	2.0135	0.0024	0.0041	0.0012
11.2	3	8.1366	0.0030	0.0014	0.0004
11.2	5	11.5635	0.0042	0.0020	0.0004
11.3	2	2.0133	0.0021	0.0032	0.0011
11.3	3	8.1272	0.0025	0.0012	0.0003
11.3	5	11.5653	0.0059	0.0029	0.0005
11.4	2	2.0017	0.0017	0.0031	0.0009
11.4	3	8.1301	0.0029	0.0016	0.0004
11.4	5	11.5488	0.0050	0.0024	0.0004
11.5	2	2.0150	0.0021	0.0041	0.0010
11.5	3	8.1319	0.0024	0.0011	0.0003
11.5	5	11.5604	0.0042	0.0020	0.0004
11.6	2	2.0118	0.0021	0.0035	0.0011
11.6	3	8.1317	0.0023	0.0011	0.0003
11.6	5	11.5579	0.0035	0.0017	0.0003
11.7	2	2.0165	0.0020	0.0023	0.0010
11.7	3	8.1298	0.0027	0.0011	0.0003
11.7	5	11.5636	0.0044	0.0019	0.0004
11.8	2	2.0015	0.0019	0.0038	0.0009
11.8	3	8.1293	0.0023	0.0007	0.0003
11.8	5	11.5627	0.0045	0.0024	0.0004
11.9	2	2.0100	0.0023	0.0043	0.0012
11.9	3	8.1349	0.0023	0.0006	0.0003
11.9	5	11.5614	0.0044	0.0016	0.0004
11.10	2	2.0084	0.0023	0.0045	0.0012
11.10	3	8.1295	0.0022	0.0008	0.0003
11.10	5	11.5677	0.0046	0.0019	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
12.1	1	2.0227	0.0024	0.0030	0.0012
12.1	3	8.1372	0.0025	0.0010	0.0003
12.1	4	11.4601	0.0046	0.0022	0.0004
12.2	1	2.0195	0.0025	0.0034	0.0012
12.2	3	8.1354	0.0027	0.0009	0.0003
12.2	4	11.4556	0.0038	0.0017	0.0003
12.3	1	2.0258	0.0023	0.0032	0.0011
12.3	3	8.1470	0.0021	0.0005	0.0003
12.3	4	11.4557	0.0041	0.0017	0.0004
12.4	1	2.0234	0.0021	0.0024	0.0011
12.4	3	8.1398	0.0023	0.0008	0.0003
12.4	4	11.4577	0.0050	0.0026	0.0004
12.5	1	2.0208	0.0023	0.0031	0.0011
12.5	3	8.1354	0.0024	0.0008	0.0003
12.5	4	11.4560	0.0036	0.0016	0.0003
12.6	1	2.0202	0.0029	0.0048	0.0015
12.6	3	8.1457	0.0027	0.0010	0.0003
12.6	4	11.4575	0.0040	0.0017	0.0003
12.7	1	2.0287	0.0024	0.0045	0.0012
12.7	3	8.1383	0.0015	0.0004	0.0002
12.7	4	11.4615	0.0047	0.0022	0.0004
12.8	1	2.0128	0.0019	0.0030	0.0009
12.8	3	8.1379	0.0017	0.0004	0.0002
12.8	4	11.4585	0.0033	0.0013	0.0003
12.9	1	2.0230	0.0020	0.0025	0.0010
12.9	3	8.1424	0.0020	0.0008	0.0003
12.9	4	11.4626	0.0029	0.0010	0.0003
12.10	1	2.0202	0.0021	0.0033	0.0010
12.10	3	8.1433	0.0019	0.0007	0.0002
12.10	4	11.4578	0.0044	0.0020	0.0004
12.1	2	2.0183	0.0027	0.0028	0.0014
12.1	3	8.1390	0.0020	0.0006	0.0002
12.1	5	11.5773	0.0041	0.0015	0.0004
12.2	2	2.0156	0.0032	0.0045	0.0016
12.2	3	8.1333	0.0031	0.0011	0.0004
12.2	5	11.5741	0.0037	0.0014	0.0003
12.3	2	2.0157	0.0017	0.0022	0.0009
12.3	3	8.1393	0.0027	0.0012	0.0003
12.3	5	11.5688	0.0031	0.0013	0.0003
12.4	2	2.0159	0.0026	0.0032	0.0013
12.4	3	8.1395	0.0026	0.0011	0.0003
12.4	5	11.5782	0.0048	0.0025	0.0004
12.5	2	2.0088	0.0019	0.0030	0.0010
12.5	3	8.1438	0.0028	0.0011	0.0003
12.5	5	11.5783	0.0040	0.0018	0.0003
12.6	2	2.0167	0.0028	0.0039	0.0014
12.6	3	8.1411	0.0025	0.0008	0.0003
12.6	5	11.5712	0.0043	0.0018	0.0004
12.7	2	2.0178	0.0023	0.0030	0.0012
12.7	3	8.1444	0.0031	0.0017	0.0004
12.7	5	11.5664	0.0043	0.0020	0.0004
12.8	2	2.0193	0.0024	0.0033	0.0012
12.8	3	8.1404	0.0023	0.0005	0.0003
12.8	5	11.5788	0.0028	0.0011	0.0002
12.9	2	2.0231	0.0018	0.0022	0.0009
12.9	3	8.1362	0.0021	0.0008	0.0003
12.9	5	11.5797	0.0034	0.0013	0.0003
12.10	2	2.0180	0.0022	0.0031	0.0011
12.10	3	8.1421	0.0032	0.0015	0.0004
12.10	5	11.5826	0.0044	0.0018	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
13.1	1	2.0227	0.0024	0.0030	0.0012
13.1	3	8.1372	0.0025	0.0010	0.0003
13.1	4	11.4601	0.0046	0.0022	0.0004
13.2	1	2.0195	0.0025	0.0034	0.0012
13.2	3	8.1354	0.0027	0.0009	0.0003
13.2	4	11.4556	0.0038	0.0017	0.0003
13.3	1	2.0258	0.0023	0.0032	0.0011
13.3	3	8.1470	0.0021	0.0005	0.0003
13.3	4	11.4557	0.0041	0.0017	0.0004
13.4	1	2.0234	0.0021	0.0024	0.0011
13.4	3	8.1398	0.0023	0.0008	0.0003
13.4	4	11.4577	0.0050	0.0026	0.0004
13.5	1	2.0208	0.0023	0.0031	0.0011
13.5	3	8.1354	0.0024	0.0008	0.0003
13.5	4	11.4560	0.0036	0.0016	0.0003
13.6	1	2.0202	0.0029	0.0048	0.0015
13.6	3	8.1457	0.0027	0.0010	0.0003
13.6	4	11.4575	0.0040	0.0017	0.0003
13.7	1	2.0287	0.0024	0.0045	0.0012
13.7	3	8.1383	0.0015	0.0004	0.0002
13.7	4	11.4615	0.0047	0.0022	0.0004
13.8	1	2.0128	0.0019	0.0030	0.0009
13.8	3	8.1379	0.0017	0.0004	0.0002
13.8	4	11.4585	0.0033	0.0013	0.0003
13.9	1	2.0230	0.0020	0.0025	0.0010
13.9	3	8.1424	0.0020	0.0008	0.0003
13.9	4	11.4626	0.0029	0.0010	0.0003
13.10	1	2.0202	0.0021	0.0033	0.0010
13.10	3	8.1433	0.0019	0.0007	0.0002
13.10	4	11.4578	0.0044	0.0020	0.0004
13.1	2	2.0164	0.0020	0.0026	0.0010
13.1	3	8.1415	0.0021	0.0007	0.0003
13.1	5	11.5795	0.0036	0.0011	0.0003
13.2	2	2.0138	0.0019	0.0036	0.0009
13.2	3	8.1435	0.0021	0.0006	0.0003
13.2	5	11.5754	0.0045	0.0023	0.0004
13.3	2	2.0127	0.0017	0.0019	0.0009
13.3	3	8.1397	0.0023	0.0009	0.0003
13.3	5	11.5744	0.0037	0.0017	0.0003
13.4	2	2.0183	0.0022	0.0028	0.0011
13.4	3	8.1425	0.0026	0.0012	0.0003
13.4	5	11.5784	0.0042	0.0020	0.0004
13.5	2	2.0236	0.0021	0.0024	0.0010
13.5	3	8.1467	0.0026	0.0011	0.0003
13.5	5	11.5827	0.0027	0.0009	0.0002
13.6	2	2.0117	0.0020	0.0024	0.0010
13.6	3	8.1379	0.0028	0.0010	0.0003
13.6	5	11.5852	0.0034	0.0015	0.0003
13.7	2	2.0192	0.0022	0.0032	0.0011
13.7	3	8.1439	0.0029	0.0014	0.0004
13.7	5	11.5747	0.0043	0.0020	0.0004
13.8	2	2.0197	0.0017	0.0017	0.0008
13.8	3	8.1396	0.0025	0.0009	0.0003
13.8	5	11.5837	0.0037	0.0017	0.0003
13.9	2	2.0215	0.0018	0.0026	0.0009
13.9	3	8.1395	0.0026	0.0014	0.0003
13.9	5	11.5809	0.0029	0.0010	0.0003
13.10	2	2.0158	0.0026	0.0047	0.0013
13.10	3	8.1398	0.0026	0.0010	0.0003
13.10	5	11.5812	0.0040	0.0018	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
14.1	1	2.0227	0.0024	0.0030	0.0012
14.1	3	8.1372	0.0025	0.0010	0.0003
14.1	4	11.4601	0.0046	0.0022	0.0004
14.2	1	2.0195	0.0025	0.0034	0.0012
14.2	3	8.1354	0.0027	0.0009	0.0003
14.2	4	11.4556	0.0038	0.0017	0.0003
14.3	1	2.0258	0.0023	0.0032	0.0011
14.3	3	8.1470	0.0021	0.0005	0.0003
14.3	4	11.4557	0.0041	0.0017	0.0004
14.4	1	2.0234	0.0021	0.0024	0.0011
14.4	3	8.1398	0.0023	0.0008	0.0003
14.4	4	11.4577	0.0050	0.0026	0.0004
14.5	1	2.0208	0.0023	0.0031	0.0011
14.5	3	8.1354	0.0024	0.0008	0.0003
14.5	4	11.4560	0.0036	0.0016	0.0003
14.6	1	2.0202	0.0029	0.0048	0.0015
14.6	3	8.1457	0.0027	0.0010	0.0003
14.6	4	11.4575	0.0040	0.0017	0.0003
14.7	1	2.0287	0.0024	0.0045	0.0012
14.7	3	8.1383	0.0015	0.0004	0.0002
14.7	4	11.4615	0.0047	0.0022	0.0004
14.8	1	2.0128	0.0019	0.0030	0.0009
14.8	3	8.1379	0.0017	0.0004	0.0002
14.8	4	11.4585	0.0033	0.0013	0.0003
14.9	1	2.0230	0.0020	0.0025	0.0010
14.9	3	8.1424	0.0020	0.0008	0.0003
14.9	4	11.4626	0.0029	0.0010	0.0003
14.10	1	2.0202	0.0021	0.0033	0.0010
14.10	3	8.1433	0.0019	0.0007	0.0002
14.10	4	11.4578	0.0044	0.0020	0.0004
14.1	2	2.0260	0.0019	0.0023	0.0010
14.1	3	8.1414	0.0022	0.0007	0.0003
14.1	5	11.5939	0.0041	0.0019	0.0004
14.2	2	2.0223	0.0024	0.0028	0.0012
14.2	3	8.1427	0.0017	0.0006	0.0002
14.2	5	11.5880	0.0029	0.0012	0.0003
14.3	2	2.0186	0.0021	0.0032	0.0010
14.3	3	8.1399	0.0019	0.0006	0.0002
14.3	5	11.5783	0.0037	0.0014	0.0003
14.4	2	2.0190	0.0023	0.0040	0.0011
14.4	3	8.1438	0.0029	0.0011	0.0004
14.4	5	11.5853	0.0027	0.0008	0.0002
14.5	2	2.0144	0.0023	0.0033	0.0011
14.5	3	8.1412	0.0025	0.0008	0.0003
14.5	5	11.5786	0.0043	0.0016	0.0004
14.6	2	2.0217	0.0018	0.0029	0.0009
14.6	3	8.1400	0.0024	0.0008	0.0003
14.6	5	11.5818	0.0043	0.0021	0.0004
14.7	2	2.0282	0.0025	0.0034	0.0012
14.7	3	8.1459	0.0030	0.0013	0.0004
14.7	5	11.5811	0.0029	0.0009	0.0002
14.8	2	2.0227	0.0020	0.0025	0.0010
14.8	3	8.1377	0.0021	0.0007	0.0003
14.8	5	11.5800	0.0026	0.0010	0.0002
14.9	2	2.0110	0.0022	0.0031	0.0011
14.9	3	8.1346	0.0023	0.0009	0.0003
14.9	5	11.5866	0.0041	0.0020	0.0004
14.10	2	2.0150	0.0021	0.0026	0.0011
14.10	3	8.1346	0.0019	0.0006	0.0002
14.10	5	11.5802	0.0039	0.0014	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
15.1	1	2.0044	0.0030	0.0064	0.0015
15.1	4	8.1377	0.0032	0.0015	0.0004
15.1	4	11.4528	0.0047	0.0022	0.0004
15.2	1	2.0037	0.0022	0.0034	0.0011
15.2	3	8.1363	0.0025	0.0010	0.0003
15.2	4	11.4504	0.0049	0.0027	0.0004
15.3	1	2.0027	0.0020	0.0028	0.0010
15.3	3	8.1351	0.0022	0.0008	0.0003
15.3	4	11.4594	0.0055	0.0028	0.0005
15.4	1	2.0083	0.0027	0.0054	0.0014
15.4	3	8.1386	0.0030	0.0011	0.0004
15.4	4	11.4520	0.0047	0.0021	0.0004
15.5	1	2.0047	0.0023	0.0045	0.0011
15.5	3	8.1308	0.0031	0.0014	0.0004
15.5	4	11.4484	0.0056	0.0032	0.0005
15.6	1	2.0005	0.0021	0.0033	0.0010
15.6	3	8.1348	0.0027	0.0014	0.0003
15.6	4	11.4564	0.0049	0.0026	0.0004
15.7	1	2.0073	0.0021	0.0018	0.0010
15.7	3	8.1339	0.0035	0.0021	0.0004
15.7	4	11.4517	0.0041	0.0024	0.0004
15.8	1	2.0041	0.0025	0.0040	0.0013
15.8	3	8.1383	0.0025	0.0010	0.0003
15.8	4	11.4627	0.0059	0.0036	0.0005
15.9	1	2.0033	0.0029	0.0062	0.0014
15.9	3	8.1400	0.0032	0.0014	0.0004
15.9	4	11.4600	0.0044	0.0019	0.0004
15.10	1	2.0006	0.0023	0.0032	0.0011
15.10	3	8.1409	0.0027	0.0015	0.0003
15.10	4	11.4682	0.0061	0.0041	0.0005
15.1	2	2.0084	0.0025	0.0043	0.0013
15.1	3	8.1356	0.0031	0.0014	0.0004
15.1	5	11.5776	0.0039	0.0017	0.0003
15.2	2	2.0060	0.0018	0.0032	0.0009
15.2	3	8.1329	0.0023	0.0011	0.0003
15.2	5	11.5712	0.0042	0.0019	0.0004
15.3	2	2.0080	0.0020	0.0029	0.0010
15.3	3	8.1348	0.0022	0.0008	0.0003
15.3	5	11.5721	0.0046	0.0021	0.0004
15.4	2	2.0127	0.0030	0.0065	0.0015
15.4	3	8.1355	0.0028	0.0012	0.0003
15.4	5	11.5868	0.0050	0.0022	0.0004
15.5	2	2.0088	0.0024	0.0031	0.0012
15.5	3	8.1324	0.0035	0.0013	0.0004
15.5	5	11.5788	0.0062	0.0035	0.0005
15.6	2	2.0080	0.0019	0.0046	0.0010
15.6	3	8.1321	0.0026	0.0012	0.0003
15.6	5	11.5754	0.0045	0.0021	0.0004
15.7	2	2.0101	0.0016	0.0031	0.0008
15.7	3	8.1294	0.0032	0.0014	0.0004
15.7	5	11.5717	0.0041	0.0020	0.0004
15.8	2	2.0070	0.0022	0.0034	0.0011
15.8	3	8.1363	0.0024	0.0012	0.0003
15.8	5	11.5797	0.0059	0.0032	0.0005
15.9	2	2.0085	0.0019	0.0021	0.0009
15.9	3	8.1362	0.0031	0.0008	0.0004
15.9	5	11.5750	0.0061	0.0035	0.0005
15.10	2	2.0024	0.0016	0.0026	0.0008
15.10	3	8.1377	0.0026	0.0017	0.0003
15.10	5	11.5843	0.0048	0.0024	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
16.1	1	2.0220	0.0021	0.0040	0.0010
16.1	3	8.1314	0.0026	0.0011	0.0003
16.1	4	11.4694	0.0056	0.0035	0.0005
16.2	1	2.0218	0.0018	0.0028	0.0009
16.2	3	8.1370	0.0026	0.0015	0.0003
16.2	4	11.4566	0.0047	0.0026	0.0004
16.3	1	2.0298	0.0025	0.0042	0.0012
16.3	3	8.1375	0.0025	0.0013	0.0003
16.3	4	11.4512	0.0044	0.0022	0.0004
16.4	1	2.0140	0.0017	0.0017	0.0008
16.4	3	8.1366	0.0026	0.0010	0.0003
16.4	4	11.4553	0.0049	0.0026	0.0004
16.5	1	2.0206	0.0019	0.0028	0.0009
16.5	3	8.1337	0.0021	0.0006	0.0003
16.5	4	11.4673	0.0060	0.0034	0.0005
16.6	1	2.0117	0.0024	0.0031	0.0012
16.6	3	8.1334	0.0024	0.0009	0.0003
16.6	4	11.4566	0.0047	0.0023	0.0004
16.7	1	2.0247	0.0017	0.0025	0.0009
16.7	3	8.1435	0.0031	0.0016	0.0004
16.7	4	11.4510	0.0051	0.0025	0.0004
16.8	1	2.0173	0.0020	0.0029	0.0010
16.8	3	8.1356	0.0030	0.0015	0.0004
16.8	4	11.4601	0.0050	0.0027	0.0004
16.9	1	2.0229	0.0022	0.0031	0.0011
16.9	3	8.1276	0.0029	0.0013	0.0004
16.9	4	11.4693	0.0049	0.0028	0.0004
16.10	1	2.0232	0.0026	0.0041	0.0013
16.10	3	8.1430	0.0024	0.0008	0.0003
16.10	4	11.4448	0.0039	0.0021	0.0003
16.1	2	2.0286	0.0026	0.0040	0.0013
16.1	3	8.1308	0.0027	0.0013	0.0003
16.1	5	11.5714	0.0042	0.0019	0.0004
16.2	2	2.0241	0.0019	0.0033	0.0009
16.2	3	8.1342	0.0025	0.0013	0.0003
16.2	5	11.5692	0.0045	0.0023	0.0004
16.3	2	2.0290	0.0022	0.0039	0.0011
16.3	3	8.1399	0.0025	0.0011	0.0003
16.3	5	11.5585	0.0051	0.0026	0.0004
16.4	2	2.0168	0.0022	0.0039	0.0011
16.4	3	8.1374	0.0025	0.0008	0.0003
16.4	5	11.5622	0.0048	0.0026	0.0004
16.5	2	2.0254	0.0019	0.0032	0.0010
16.5	3	8.1339	0.0022	0.0007	0.0003
16.5	5	11.5844	0.0041	0.0018	0.0004
16.6	2	2.0223	0.0029	0.0054	0.0014
16.6	3	8.1312	0.0024	0.0010	0.0003
16.6	5	11.5808	0.0036	0.0016	0.0003
16.7	2	2.0273	0.0021	0.0027	0.0010
16.7	3	8.1419	0.0029	0.0014	0.0004
16.7	5	11.5724	0.0037	0.0015	0.0003
16.8	2	2.0248	0.0018	0.0022	0.0009
16.8	3	8.1363	0.0028	0.0012	0.0003
16.8	5	11.5724	0.0045	0.0021	0.0004
16.9	2	2.0203	0.0026	0.0044	0.0013
16.9	3	8.1279	0.0029	0.0014	0.0004
16.9	5	11.5673	0.0050	0.0026	0.0004
16.10	2	2.0244	0.0029	0.0053	0.0014
16.10	3	8.1424	0.0026	0.0009	0.0003
16.10	5	11.5569	0.0043	0.0023	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
17.1	1	2.0216	0.0024	0.0036	0.0012
17.1	3	8.1402	0.0029	0.0011	0.0004
17.1	4	11.4594	0.0042	0.0017	0.0004
17.2	1	2.0095	0.0024	0.0036	0.0012
17.2	3	8.1348	0.0027	0.0010	0.0003
17.2	4	11.4525	0.0042	0.0018	0.0004
17.3	1	2.0191	0.0029	0.0061	0.0015
17.3	3	8.1423	0.0029	0.0013	0.0004
17.3	4	11.4619	0.0046	0.0024	0.0004
17.4	1	2.0087	0.0018	0.0022	0.0009
17.4	3	8.1409	0.0029	0.0011	0.0004
17.4	4	11.4490	0.0040	0.0016	0.0003
17.5	1	2.0137	0.0021	0.0030	0.0010
17.5	3	8.1409	0.0026	0.0010	0.0003
17.5	4	11.4450	0.0039	0.0018	0.0003
17.6	1	2.0088	0.0017	0.0021	0.0008
17.6	3	8.1365	0.0026	0.0010	0.0003
17.6	4	11.4527	0.0040	0.0017	0.0003
17.7	1	2.0058	0.0016	0.0019	0.0008
17.7	3	8.1382	0.0020	0.0007	0.0002
17.7	4	11.4477	0.0036	0.0015	0.0003
17.8	1	2.0105	0.0020	0.0012	0.0010
17.8	3	8.1383	0.0030	0.0013	0.0004
17.8	4	11.4451	0.0033	0.0012	0.0003
17.9	1	2.0130	0.0022	0.0035	0.0011
17.9	3	8.1386	0.0027	0.0013	0.0003
17.9	4	11.4610	0.0045	0.0024	0.0004
17.10	1	2.0077	0.0016	0.0019	0.0008
17.10	3	8.1363	0.0021	0.0007	0.0003
17.10	4	11.4506	0.0046	0.0025	0.0004
17.1	2	2.0234	0.0029	0.0048	0.0014
17.1	3	8.1431	0.0030	0.0011	0.0004
17.1	5	11.5676	0.0053	0.0027	0.0005
17.2	2	2.0221	0.0028	0.0053	0.0014
17.2	3	8.1339	0.0028	0.0011	0.0003
17.2	5	11.5681	0.0053	0.0028	0.0005
17.3	2	2.0275	0.0033	0.0069	0.0017
17.3	3	8.1430	0.0031	0.0015	0.0004
17.3	5	11.5767	0.0047	0.0026	0.0004
17.4	2	2.0141	0.0025	0.0046	0.0012
17.4	3	8.1406	0.0030	0.0013	0.0004
17.4	5	11.5799	0.0050	0.0028	0.0004
17.5	2	2.0187	0.0030	0.0067	0.0015
17.5	3	8.1398	0.0026	0.0010	0.0003
17.5	5	11.5739	0.0051	0.0029	0.0004
17.6	2	2.0121	0.0027	0.0052	0.0014
17.6	3	8.1353	0.0026	0.0011	0.0003
17.6	5	11.5658	0.0056	0.0031	0.0005
17.7	2	2.0159	0.0022	0.0036	0.0011
17.7	3	8.1383	0.0021	0.0007	0.0003
17.7	5	11.5611	0.0055	0.0033	0.0005
17.8	2	2.0234	0.0026	0.0050	0.0013
17.8	3	8.1376	0.0029	0.0012	0.0004
17.8	5	11.5488	0.0053	0.0028	0.0005
17.9	2	2.0193	0.0025	0.0045	0.0012
17.9	3	8.1389	0.0028	0.0014	0.0003
17.9	5	11.5758	0.0043	0.0021	0.0004
17.10	2	2.0136	0.0023	0.0041	0.0012
17.10	3	8.1367	0.0021	0.0006	0.0003
17.10	5	11.5774	0.0051	0.0028	0.0004

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
18.1	1	2.0143	0.0023	0.0032	0.0011
18.1	3	8.1483	0.0017	0.0005	0.0002
18.1	4	11.4724	0.0046	0.0024	0.0004
18.2	1	2.0217	0.0018	0.0027	0.0009
18.2	3	8.1469	0.0025	0.0013	0.0003
18.2	4	11.4858	0.0051	0.0028	0.0004
18.3	1	2.0254	0.0026	0.0050	0.0013
18.3	3	8.1511	0.0034	0.0018	0.0004
18.3	4	11.4806	0.0053	0.0028	0.0005
18.4	1	2.0101	0.0021	0.0023	0.0010
18.4	3	8.1429	0.0026	0.0010	0.0003
18.4	4	11.4702	0.0046	0.0020	0.0004
18.5	1	2.0230	0.0020	0.0026	0.0010
18.5	3	8.1442	0.0021	0.0009	0.0003
18.5	4	11.4735	0.0044	0.0022	0.0004
18.6	1	2.0157	0.0025	0.0040	0.0012
18.6	3	8.1434	0.0028	0.0011	0.0003
18.6	4	11.4766	0.0046	0.0024	0.0004
18.7	1	2.0232	0.0025	0.0036	0.0012
18.7	3	8.1439	0.0015	0.0004	0.0002
18.7	4	11.4788	0.0047	0.0025	0.0004
18.8	1	2.0134	0.0027	0.0043	0.0014
18.8	3	8.1435	0.0026	0.0013	0.0003
18.8	4	11.4681	0.0041	0.0019	0.0004
18.9	1	2.0155	0.0022	0.0042	0.0011
18.9	3	8.1482	0.0022	0.0007	0.0003
18.9	4	11.4746	0.0058	0.0039	0.0005
18.10	1	2.0122	0.0039	0.0103	0.0020
18.10	3	8.1461	0.0025	0.0010	0.0003
18.10	4	11.4636	0.0053	0.0031	0.0005
18.1	2	2.0213	0.0022	0.0032	0.0011
18.1	3	8.1488	0.0019	0.0006	0.0002
18.1	5	11.5907	0.0036	0.0014	0.0003
18.2	2	2.0265	0.0018	0.0021	0.0009
18.2	3	8.1489	0.0027	0.0017	0.0003
18.2	5	11.5944	0.0035	0.0014	0.0003
18.3	2	2.0262	0.0024	0.0036	0.0012
18.3	3	8.1501	0.0034	0.0018	0.0004
18.3	5	11.6021	0.0035	0.0014	0.0003
18.4	2	2.0155	0.0017	0.0013	0.0009
18.4	3	8.1425	0.0026	0.0007	0.0003
18.4	5	11.5975	0.0044	0.0017	0.0004
18.5	2	2.0234	0.0020	0.0034	0.0010
18.5	3	8.1439	0.0022	0.0008	0.0003
18.5	5	11.5938	0.0039	0.0017	0.0003
18.6	2	2.0256	0.0024	0.0041	0.0012
18.6	3	8.1421	0.0029	0.0014	0.0004
18.6	5	11.5932	0.0032	0.0011	0.0003
18.7	2	2.0280	0.0026	0.0044	0.0013
18.7	3	8.1438	0.0015	0.0004	0.0002
18.7	5	11.5821	0.0046	0.0021	0.0004
18.8	2	2.0259	0.0026	0.0045	0.0013
18.8	3	8.1432	0.0028	0.0011	0.0003
18.8	5	11.5842	0.0043	0.0022	0.0004
18.9	2	2.0152	0.0027	0.0046	0.0014
18.9	3	8.1474	0.0023	0.0008	0.0003
18.9	5	11.5908	0.0041	0.0023	0.0004
18.10	2	2.0110	0.0036	0.0083	0.0018
18.10	3	8.1461	0.0028	0.0015	0.0003
18.10	5	11.5945	0.0036	0.0016	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
19.1	1	2.0190	0.0028	0.0057	0.0014
19.1	3	8.1363	0.0024	0.0009	0.0003
19.1	4	11.4576	0.0044	0.0021	0.0004
19.2	1	2.0177	0.0025	0.0037	0.0012
19.2	3	8.1395	0.0034	0.0018	0.0004
19.2	4	11.4614	0.0047	0.0022	0.0004
19.3	1	2.0107	0.0028	0.0053	0.0014
19.3	3	8.1374	0.0033	0.0016	0.0004
19.3	4	11.4657	0.0054	0.0028	0.0005
19.4	1	2.0179	0.0017	0.0026	0.0009
19.4	3	8.1359	0.0020	0.0007	0.0002
19.4	4	11.4948	0.0049	0.0026	0.0004
19.5	1	2.0119	0.0022	0.0043	0.0011
19.5	3	8.1353	0.0028	0.0014	0.0004
19.5	4	11.4847	0.0048	0.0025	0.0004
19.6	1	2.0220	0.0024	0.0043	0.0012
19.6	3	8.1387	0.0025	0.0011	0.0003
19.6	4	11.4901	0.0045	0.0028	0.0004
19.7	1	2.0158	0.0033	0.0059	0.0017
19.7	3	8.1337	0.0031	0.0014	0.0004
19.7	4	11.4475	0.0045	0.0021	0.0004
19.8	1	2.0145	0.0019	0.0026	0.0009
19.8	3	8.1348	0.0028	0.0012	0.0003
19.8	4	11.4652	0.0047	0.0023	0.0004
19.9	1	2.0103	0.0020	0.0019	0.0010
19.9	3	8.1380	0.0026	0.0013	0.0003
19.9	4	11.5095	0.0050	0.0029	0.0004
19.10	1	2.0102	0.0026	0.0044	0.0013
19.10	3	8.1350	0.0020	0.0003	0.0002
19.10	4	11.4604	0.0058	0.0036	0.0005
19.1	2	2.0279	0.0033	0.0072	0.0016
19.1	3	8.1360	0.0024	0.0009	0.0003
19.1	5	11.5791	0.0048	0.0024	0.0004
19.2	2	2.0225	0.0021	0.0031	0.0011
19.2	3	8.1366	0.0034	0.0017	0.0004
19.2	5	11.5766	0.0041	0.0019	0.0004
19.3	2	2.0162	0.0024	0.0023	0.0012
19.3	3	8.1346	0.0032	0.0016	0.0004
19.3	5	11.5705	0.0045	0.0023	0.0004
19.4	2	2.0173	0.0016	0.0020	0.0008
19.4	3	8.1351	0.0020	0.0007	0.0002
19.4	5	11.5803	0.0031	0.0012	0.0003
19.5	2	2.0233	0.0019	0.0023	0.0009
19.5	3	8.1349	0.0027	0.0011	0.0003
19.5	5	11.5818	0.0032	0.0013	0.0003
19.6	2	2.0250	0.0020	0.0033	0.0010
19.6	3	8.1389	0.0026	0.0010	0.0003
19.6	5	11.5783	0.0025	0.0017	0.0002
19.7	2	2.0217	0.0022	0.0035	0.0011
19.7	3	8.1349	0.0031	0.0014	0.0004
19.7	5	11.5676	0.0070	0.0046	0.0006
19.8	2	2.0164	0.0017	0.0022	0.0008
19.8	3	8.1327	0.0028	0.0012	0.0003
19.8	5	11.5821	0.0031	0.0013	0.0003
19.9	2	2.0147	0.0019	0.0025	0.0009
19.9	3	8.1356	0.0027	0.0012	0.0003
19.9	5	11.5864	0.0022	0.0007	0.0002
19.10	2	2.0141	0.0021	0.0029	0.0011
19.10	3	8.1351	0.0020	0.0003	0.0003
19.10	5	11.5881	0.0036	0.0016	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
20.1	1	2.0178	0.0027	0.0059	0.0013
20.1	3	8.1403	0.0027	0.0014	0.0003
20.1	4	11.4594	0.0034	0.0014	0.0003
20.2	1	2.0200	0.0032	0.0062	0.0016
20.2	3	8.1396	0.0028	0.0013	0.0003
20.2	4	11.4485	0.0054	0.0030	0.0005
20.3	1	2.0192	0.0034	0.0078	0.0017
20.3	3	8.1437	0.0030	0.0015	0.0004
20.3	4	11.4580	0.0035	0.0015	0.0003
20.4	1	2.0182	0.0033	0.0078	0.0016
20.4	3	8.1475	0.0042	0.0029	0.0005
20.4	4	11.4640	0.0039	0.0018	0.0003
20.5	1	2.0076	0.0024	0.0040	0.0012
20.5	3	8.1358	0.0031	0.0015	0.0004
20.5	4	11.4613	0.0037	0.0016	0.0003
20.6	1	2.0026	0.0048	0.0154	0.0025
20.6	3	8.1435	0.0032	0.0015	0.0004
20.6	4	11.4608	0.0039	0.0020	0.0003
20.7	1	2.0126	0.0034	0.0068	0.0017
20.7	3	8.1372	0.0030	0.0015	0.0004
20.7	4	11.4590	0.0037	0.0015	0.0003
20.8	1	2.0097	0.0035	0.0076	0.0018
20.8	3	8.1380	0.0026	0.0010	0.0003
20.8	4	11.4613	0.0036	0.0013	0.0003
20.9	1	2.0112	0.0036	0.0086	0.0018
20.9	3	8.1386	0.0027	0.0012	0.0003
20.9	4	11.4504	0.0041	0.0020	0.0004
20.10	1	2.0186	0.0029	0.0063	0.0014
20.10	3	8.1390	0.0026	0.0011	0.0003
20.10	4	11.4548	0.0043	0.0022	0.0004
20.1	2	2.0040	0.0055	0.0197	0.0028
20.1	3	8.1392	0.0028	0.0014	0.0003
20.1	5	11.5690	0.0045	0.0021	0.0004
20.2	2	2.0300	0.0030	0.0051	0.0015
20.2	3	8.1425	0.0026	0.0010	0.0003
20.2	5	11.5841	0.0060	0.0035	0.0005
20.3	2	2.0203	0.0029	0.0050	0.0014
20.3	3	8.1405	0.0029	0.0012	0.0004
20.3	5	11.5732	0.0054	0.0030	0.0005
20.4	2	2.0194	0.0031	0.0060	0.0016
20.4	3	8.1490	0.0040	0.0025	0.0005
20.4	5	11.5767	0.0049	0.0026	0.0004
20.5	2	2.0112	0.0027	0.0045	0.0013
20.5	3	8.1341	0.0031	0.0014	0.0004
20.5	5	11.5800	0.0041	0.0022	0.0004
20.6	2	2.0094	0.0032	0.0063	0.0016
20.6	3	8.1427	0.0031	0.0013	0.0004
20.6	5	11.5833	0.0027	0.0008	0.0002
20.7	2	2.0130	0.0033	0.0072	0.0017
20.7	3	8.1368	0.0029	0.0012	0.0004
20.7	5	11.5626	0.0051	0.0029	0.0004
20.8	2	2.0203	0.0041	0.0104	0.0020
20.8	3	8.1377	0.0025	0.0009	0.0003
20.8	5	11.5812	0.0043	0.0021	0.0004
20.9	2	2.0106	0.0041	0.0106	0.0020
20.9	3	8.1377	0.0025	0.0011	0.0003
20.9	5	11.5816	0.0048	0.0025	0.0004
20.10	2	2.0144	0.0028	0.0050	0.0014
20.10	3	8.1385	0.0028	0.0014	0.0003
20.10	5	11.5802	0.0054	0.0030	0.0005

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
21.1	1	2.0054	0.0034	0.0081	0.0017
21.1	3	8.1295	0.0026	0.0014	0.0003
21.1	4	11.4475	0.0065	0.0054	0.0006
21.2	1	2.0065	0.0047	0.0140	0.0024
21.2	3	8.1233	0.0030	0.0014	0.0004
21.2	4	11.4472	0.0051	0.0029	0.0004
21.3	1	1.9966	0.0055	0.0188	0.0028
21.3	3	8.1206	0.0055	0.0046	0.0007
21.3	4	11.4215	0.0257	0.0397	0.0024
21.4	1	2.0085	0.0040	0.0099	0.0020
21.4	3	8.1250	0.0026	0.0011	0.0003
21.4	4	11.4592	0.0053	0.0032	0.0005
21.5	1	2.0005	0.0041	0.0115	0.0021
21.5	3	8.1205	0.0051	0.0040	0.0006
21.5	4	11.4451	0.0058	0.0034	0.0005
21.6	1	2.0001	0.0047	0.0140	0.0024
21.6	3	8.1225	0.0023	0.0009	0.0003
21.6	4	11.4378	0.0072	0.0052	0.0006
21.7	1	2.0062	0.0031	0.0070	0.0016
21.7	3	8.1303	0.0039	0.0027	0.0005
21.7	4	11.4478	0.0039	0.0017	0.0003
21.8	1	2.0006	0.0043	0.0112	0.0022
21.8	3	8.1151	0.0042	0.0030	0.0005
21.8	4	11.4420	0.0052	0.0030	0.0005
21.9	1	2.0012	0.0035	0.0072	0.0018
21.9	3	8.1185	0.0022	0.0010	0.0003
21.9	4	11.4500	0.0053	0.0030	0.0005
21.10	1	1.9985	0.0036	0.0089	0.0018
21.10	3	8.1225	0.0047	0.0034	0.0006
21.10	4	11.4659	0.0070	0.0051	0.0006
21.1	2	2.0036	0.0043	0.0117	0.0021
21.1	3	8.1281	0.0028	0.0014	0.0003
21.1	5	11.5701	0.0066	0.0051	0.0006
21.2	2	1.9988	0.0054	0.0190	0.0027
21.2	3	8.1231	0.0024	0.0009	0.0003
21.2	5	11.5730	0.0049	0.0026	0.0004
21.3	2	1.9845	0.0061	0.0244	0.0031
21.3	3	8.1168	0.0048	0.0036	0.0006
21.3	5	11.5769	0.0076	0.0055	0.0007
21.4	2	2.0114	0.0042	0.0117	0.0021
21.4	3	8.1253	0.0026	0.0013	0.0003
21.4	5	11.5710	0.0036	0.0012	0.0003
21.5	2	2.0062	0.0044	0.0130	0.0022
21.5	3	8.1267	0.0050	0.0041	0.0006
21.5	5	11.5729	0.0076	0.0055	0.0007
21.6	2	1.9753	0.0062	0.0251	0.0032
21.6	3	8.1222	0.0024	0.0010	0.0003
21.6	5	11.5630	0.0082	0.0065	0.0007
21.7	2	1.9988	0.0046	0.0133	0.0023
21.7	3	8.1348	0.0043	0.0032	0.0005
21.7	5	11.5753	0.0047	0.0023	0.0004
21.8	2	1.9855	0.0059	0.0214	0.0030
21.8	3	8.1186	0.0033	0.0017	0.0004
21.8	5	11.5620	0.0068	0.0049	0.0006
21.9	2	1.9991	0.0048	0.0128	0.0024
21.9	3	8.1202	0.0023	0.0011	0.0003
21.9	5	11.5661	0.0057	0.0031	0.0005
21.10	2	1.9877	0.0062	0.0253	0.0032
21.10	3	8.1257	0.0046	0.0033	0.0006
21.10	5	11.5659	0.0036	0.0016	0.0003

Measurement Number	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
22.1	1	2.0191	0.0028	0.0044	0.0014
22.1	3	8.1387	0.0021	0.0008	0.0003
22.1	4	11.4594	0.0038	0.0018	0.0003
22.2	1	2.0170	0.0027	0.0049	0.0014
22.2	3	8.1366	0.0026	0.0011	0.0003
22.2	4	11.4626	0.0047	0.0022	0.0004
22.3	1	2.0108	0.0022	0.0028	0.0011
22.3	3	8.1428	0.0029	0.0012	0.0004
22.3	4	11.4911	0.0052	0.0032	0.0004
22.4	1	2.0141	0.0025	0.0046	0.0013
22.4	3	8.1439	0.0030	0.0015	0.0004
22.4	4	11.4631	0.0046	0.0022	0.0004
22.5	1	2.0161	0.0021	0.0032	0.0010
22.5	3	8.1371	0.0031	0.0014	0.0004
22.5	4	11.4586	0.0043	0.0021	0.0004
22.6	1	2.0125	0.0023	0.0043	0.0012
22.6	3	8.1429	0.0037	0.0020	0.0005
22.6	4	11.4735	0.0050	0.0029	0.0004
22.7	1	2.0250	0.0030	0.0052	0.0015
22.7	3	8.1396	0.0024	0.0007	0.0003
22.7	4	11.4797	0.0059	0.0036	0.0005
22.8	1	2.0262	0.0030	0.0050	0.0015
22.8	3	8.1407	0.0037	0.0020	0.0005
22.8	4	11.5062	0.0053	0.0029	0.0005
22.9	1	2.0229	0.0025	0.0042	0.0012
22.9	3	8.1369	0.0029	0.0014	0.0004
22.9	4	11.4752	0.0045	0.0018	0.0004
22.10	1	2.0199	0.0024	0.0038	0.0012
22.10	3	8.1338	0.0029	0.0014	0.0004
22.10	4	11.4896	0.0051	0.0026	0.0004
22.1	2	2.0193	0.0022	0.0040	0.0011
22.1	3	8.1389	0.0019	0.0008	0.0002
22.1	5	11.5848	0.0039	0.0019	0.0003
22.2	2	2.0173	0.0026	0.0048	0.0013
22.2	3	8.1362	0.0026	0.0010	0.0003
22.2	5	11.5779	0.0044	0.0020	0.0004
22.3	2	2.0127	0.0020	0.0024	0.0010
22.3	3	8.1403	0.0028	0.0012	0.0003
22.3	5	11.5927	0.0035	0.0017	0.0003
22.4	2	2.0156	0.0019	0.0042	0.0009
22.4	3	8.1423	0.0028	0.0013	0.0003
22.4	5	11.5830	0.0032	0.0011	0.0003
22.5	2	2.0194	0.0017	0.0024	0.0009
22.5	3	8.1354	0.0029	0.0013	0.0004
22.5	5	11.5841	0.0060	0.0036	0.0005
22.6	2	2.0133	0.0027	0.0033	0.0013
22.6	3	8.1387	0.0035	0.0016	0.0004
22.6	5	11.5795	0.0046	0.0022	0.0004
22.7	2	2.0229	0.0019	0.0028	0.0010
22.7	3	8.1380	0.0023	0.0006	0.0003
22.7	5	11.5882	0.0038	0.0014	0.0003
22.8	2	2.0336	0.0022	0.0036	0.0011
22.8	3	8.1385	0.0035	0.0017	0.0004
22.8	5	11.5878	0.0033	0.0010	0.0003
22.9	2	2.0197	0.0019	0.0025	0.0009
22.9	3	8.1366	0.0027	0.0012	0.0003
22.9	5	11.5849	0.0033	0.0015	0.0003
22.10	2	2.0248	0.0022	0.0020	0.0011
22.10	3	8.1332	0.0028	0.0015	0.0003
22.10	5	11.5863	0.0035	0.0012	0.0003

ENCLOSURE J

Estimated Modal Parameters (Damaged Mast)

Measurement Number	Damage State	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
4.1	1	1	2.0017	0.0017	0.0017	0.0009
4.1	1	3	7.4477	0.0024	0.0010	0.0003
4.1	1	4	10.8242	0.0044	0.0021	0.0004
4.1	2	1	2.0281	0.0021	0.0028	0.0010
4.1	2	3	7.5256	0.0028	0.0012	0.0004
4.1	2	4	11.4894	0.0041	0.0019	0.0004
4.1	5	1	1.9845	0.0025	0.0040	0.0013
4.1	5	3	6.4521	0.0046	0.0038	0.0007
4.1	5	4	10.4237	0.0054	0.0037	0.0005
4.1	6	1	2.0157	0.0022	0.0048	0.0011
4.1	6	3	6.5345	0.0053	0.0048	0.0008
4.1	6	4	11.4286	0.0073	0.0053	0.0006
4.1	9	1	2.0221	0.0027	0.0038	0.0013
4.1	9	3	8.1357	0.0028	0.0010	0.0003
4.1	9	4	11.4822	0.0037	0.0014	0.0003
4.1	10	1	2.0287	0.0024	0.0039	0.0012
4.1	10	3	8.0940	0.0018	0.0005	0.0002
4.1	10	4	11.4662	0.0044	0.0020	0.0004
4.1	11	1	2.0352	0.0022	0.0028	0.0011
4.1	11	3	8.1291	0.0025	0.0009	0.0003
4.1	11	4	11.4839	0.0055	0.0027	0.0005
4.1	1	2	2.0258	0.0025	0.0033	0.0012
4.1	1	3	7.4406	0.0044	0.0029	0.0006
4.1	1	5	11.3466	0.0089	0.0087	0.0008
4.1	2	2	2.0145	0.0030	0.0051	0.0015
4.1	2	3	7.5286	0.0029	0.0012	0.0004
4.1	2	5	10.8375	0.0046	0.0024	0.0004
4.1	5	2	2.0222	0.0020	0.0029	0.0010
4.1	5	3	6.4139	0.0074	0.0097	0.0012
4.1	5	5	11.3844	0.0100	0.0104	0.0009
4.1	6	2	1.9797	0.0026	0.0044	0.0013
4.1	6	3	6.5585	0.0059	0.0058	0.0009
4.1	6	5	10.3491	0.0053	0.0035	0.0005
4.1	9	2	2.0312	0.0025	0.0040	0.0012
4.1	9	3	8.1345	0.0025	0.0009	0.0003
4.1	9	5	11.5983	0.0046	0.0024	0.0004
4.1	10	2	2.0384	0.0027	0.0041	0.0013
4.1	10	3	8.0955	0.0019	0.0005	0.0002
4.1	10	5	11.5683	0.0048	0.0025	0.0004
4.1	11	2	2.0316	0.0022	0.0039	0.0011
4.1	11	3	8.1304	0.0026	0.0013	0.0003
4.1	11	5	11.6159	0.0042	0.0018	0.0004

Measurement Number	Damage State	Mode Number	Frequency Mean (Hz)	Frequency Std. (Hz)	Damp. ratio Mean	Damp. ratio Std.
6.1	1	1	1.9927	0.0020	0.0035	0.0010
6.1	1	3	7.4268	0.0020	0.0007	0.0003
6.1	1	4	10.7744	0.0054	0.0033	0.0005
6.1	2	1	2.0228	0.0026	0.0045	0.0013
6.1	2	3	7.5314	0.0028	0.0018	0.0004
6.1	2	4	11.4208	0.0069	0.0051	0.0006
6.1	5	1	1.9658	0.0028	0.0045	0.0014
6.1	5	3	6.4684	0.0044	0.0038	0.0007
6.1	5	4	10.4193	0.0052	0.0035	0.0005
6.1	6	1	2.0151	0.0023	0.0035	0.0011
6.1	6	3	6.5298	0.0022	0.0009	0.0003
6.1	6	4	10.7018	0.0115	0.0148	0.0011
6.1	9	1	2.0154	0.0023	0.0041	0.0012
6.1	9	3	8.1123	0.0022	0.0007	0.0003
6.1	9	4	11.4579	0.0049	0.0024	0.0004
6.1	10	1	2.0191	0.0024	0.0036	0.0012
6.1	10	3	8.0991	0.0021	0.0007	0.0003
6.1	10	4	11.4419	0.0040	0.0021	0.0003
6.1	11	1	2.0144	0.0021	0.0025	0.0010
6.1	11	3	8.1339	0.0023	0.0008	0.0003
6.1	11	4	11.4539	0.0035	0.0013	0.0003
6.1	1	2	2.0181	0.0019	0.0020	0.0009
6.1	1	3	7.4223	0.0021	0.0008	0.0003
6.1	1	5	11.5678	0.0051	0.0029	0.0004
6.1	2	2	2.0020	0.0030	0.0058	0.0015
6.1	2	3	7.5291	0.0031	0.0018	0.0004
6.1	2	5	10.8088	0.0050	0.0028	0.0005
6.1	5	2	2.0059	0.0037	0.0081	0.0018
6.1	5	3	6.4413	0.0068	0.0086	0.0011
6.1	5	5	11.4989	0.0071	0.0062	0.0006
6.1	6	2	1.9642	0.0016	0.0021	0.0008
6.1	6	3	6.5294	0.0024	0.0011	0.0004
6.1	6	5	10.3022	0.0033	0.0014	0.0003
6.1	9	2	2.0213	0.0020	0.0028	0.0010
6.1	9	3	8.1123	0.0021	0.0008	0.0003
6.1	9	5	11.5924	0.0036	0.0028	0.0003
6.1	10	2	2.0207	0.0034	0.0030	0.0010
6.1	10	3	8.0972	0.0020	0.0008	0.0002
6.1	10	5	11.5587	0.0034	0.0012	0.0003
6.1	11	2	2.0172	0.0021	0.0031	0.0003
6.1	11	3	8.1309	0.0023	0.0007	0.0003
6.1	11	5	11.5973	0.0034	0.0031	0.0011

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